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A COLLECTION OF PASCAL PROGRAMS

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Contents

0. Preface

This is a collection of a wide variety of Pascal programs. They range in complexity from simple examples used in introductory courses to illustrate design principles and language features to intricate examples discussed in courses on algorithms and data structures. The programs, however, are grouped according to subject matter rather than complexity. Many are taken from the literature listed below, where they are explained and analyzed in detail.

The main purpose of this booklet is to provide the teacher of programming with a condensed collection of exemplary programs and thereby to exhibit a preferred style of programming using a structured language. At the same time, the booklet may serve as a guide in inventing other, perhaps similar exercises. Lastly, it may be a helpful reference to some widely used, fundamental algorithms, formulated in detail in a widely available language.

References

N. Wirth, *Systematic programming.* Prentice-Hall, Inc. 1973.

-- *Algorithms + data structures = programs.* Prentice-Hall, Inc. 1975.

1. Integer arithmetic

1. Raise integer to a positive power. Repeat reading pairs of integers, until you encounter a 0. Indicate invariant of loop.

```
PROGRAM power(input, output);
  VAR a, b: integer;

  FUNCTION power (x,n: integer): integer;
    VAR w,z,i: integer;
  BEGIN w := x; i := n; z := 1;
    WHILE i # 0 DO
    BEGIN (* z*w↑i = x↑n *)
      IF odd(i) THEN z := z*w;
      w := sqr(w); i := i DIV 2
    END;
    power := z
  END (* power *) ;

BEGIN read(a);
  WHILE a # 0 DO
  BEGIN read(b); writeln(a, b, power(a,b));
    read(a)
  END
END .
```

2. Divide an integer by a natural number, using operations of addition, subtraction, doubling and halving only. Repeat reading pairs of integers, until you encounter a 0. For each pair, print dividend, divisor, quotient, and remainder. Indicate invariant of loop.

```
PROGRAM divide(input, output);
  VAR a,b,q,r: integer;

  PROCEDURE divide (x,y: integer; VAR z,a: integer);
    VAR q,r,w: integer;
  BEGIN r := x; w := y; q := 0;
    WHILE w <= r DO w := 2*w;
    WHILE w # y DO
    BEGIN (* x = q*w + r *)  w := w DIV 2; q := 2*q;
      IF w <= r THEN
      BEGIN r := r-w; q := q+1
      END
    END ;
    z := q; a := r
  END (*divide*) ;

BEGIN read(a);
  WHILE a # 0 DO
  BEGIN read(b); divide(a,b,q,r); writeln(a, b, q, r);
    read(a)
  END
END .
```

3. Compute the greatest common divisor (gcd) and the lowest common mulliple (lcm) of two natural numbers by using addition and subtraction only. Note that gcd(m,n) * lcm(m,n) = m*n. Repeat reading pairs of integers, until you encounter a 0. For each pair, print the arguments, the gcd and the lcm. Indicate the loop invariant.

```
PROGRAM gcdlcm(input, output);
   VAR a,b,c,d: integer;

PROCEDURE gcd(x,y: integer; VAR u,v: integer);
   VAR a,b,c,d: integer;
BEGIN a := x; c := x; b := y; d := y;
   WHILE a # b DO
   BEGIN (*gcd(a,b) = gcd(x,y) AND a*d + b*c = 2*x*y*)
      IF a > b THEN
         BEGIN a := a-b; c := c+d
         END
      ELSE
         BEGIN b := b-a; d := d+c
         END
   END ;
   u := a; v := (c+d) DIV 2
END (*gcdmult*) ;

BEGIN read(a);
   WHILE a # 0 DO
   BEGIN read(b); gcd(a,b,c,d); writeln(a, b, c, d);
      read(a)
   END
END .
```

4. Compute the greatest common divisor (gcd) of two natural numbers. Use addition, subtraction, doubling and halving only.

```
PROGRAM binarygcd(output);
   VAR a,b: integer;

   FUNCTION gcd (x,y: integer): integer;
      VAR u,v,d, a,b,k: integer;
   BEGIN u := x; v := y; a := 0; b := 0;
      WHILE NOT odd(u) DO
         BEGIN u := u DIV 2; a := a+1
         END ;
      WHILE NOT odd(v) DO
         BEGIN v := v DIV 2; b := b+1
         END ;
      IF a<b THEN k := a ELSE k := b;
      d := u - v;
      WHILE d # 0 DO
      BEGIN REPEAT d := d DIV 2 UNTIL odd(d);
         IF d<0 THEN v := -d ELSE u := d;
         d := u - v
      END ;
      WHILE k>0 DO
      BEGIN u := 2*u; k := k-1
      END;
      gcd := u
   END ;
```

```
BEGIN read(a);
   WHILE a # 0 DO
   BEGIN read(b); writeln(a, b, gcd(a,b)); read(a)
   END
END .
```

5. Compute the largest integer less or equal to the square root of a given integer (due to Hoare).

```
PROGRAM isqrt(input,output);
  VAR n,a2,b2,ab,t: integer;
BEGIN read(n);
  WHILE n >= 0 DO
  BEGIN a2 := 0; ab := 0; b2 := 1; writeln(" n =", n);
    WHILE b2 <= n DO b2 := 4*b2;
    WHILE b2 # 1 DO
    BEGIN (* a2+2*ab+b2 > n, 0 <= a2 <= n, sqr(ab) = a2*b2 *)
      ab := ab DIV 2; b2 := b2 DIV 4; t := a2 + 2*ab + b2;
      IF t <= n THEN
        BEGIN a2 := t; ab := ab + b2;
        END
    END ;
    writeln(a2,ab,b2); read(n)
  END
END .
```

2. Integer arithmetic and arrays

1. Find all integers between 1 and 1000 whose squares are palindromes. Examples: 11↑2 = 121, 22↑2 = 484.

```
PROGRAM palindromes(output);
  VAR i,j,l,n,r,s: integer;
      p: boolean;
      d: ARRAY [1..10] OF integer;
BEGIN n := 0;
  REPEAT n := n+1; s := n*n; l := 0;
    REPEAT l := l+1; r := s DIV 10;
      d[l] := s - 10*r; s := r
    UNTIL s = 0;
    i := 1; j := l;
    REPEAT p := d[i]=d[j];
      i := i+1; j := j-1
    UNTIL (i>=j) OR NOT p;
    IF p THEN writeln(n,n*n)
  UNTIL n = 1000
END .
```

2. Compute and print magic squares of order 3, 5, 7, ...

```
PROGRAM magicsquare(output);
   CONST lim = 11;
   VAR i,j,x,nx,nsq,n: integer;
       m: ARRAY [1..lim,1..lim] OF integer;

   PROCEDURE getsquare;
   BEGIN x := 0; nsq := sqr(n);
      i := (n+1) DIV 2; j := n+1;
      REPEAT nx := x + n; j := j-1;
         x := x+1; m[i,j] := x;
         REPEAT i := i+1; IF i > n THEN i := 1;
                j := j+1; IF j > n THEN j := 1;
                x := x+1; m[i,j] := x
         UNTIL x = nx
      UNTIL x = nsq
   END (*getsquare*) ;

   PROCEDURE printsquare;
   BEGIN
      FOR i := 1 TO n DO
      BEGIN FOR j := 1 TO n DO write(m[i,j]: 6);
         writeln
      END ;
      writeln
   END (*printsquare*) ;

BEGIN n := 3;
   REPEAT getsquare; printsquare; n := n+2
   UNTIL n > lim
END .
```

3. Compute a table of positive and negative powers of 2. Exponents range from 1 to, say, 64. Do not truncate any digits!

```
PROGRAM powersoftwo(output);
CONST m = 31; n = 100;  (* m = n*log(2) *)
VAR exp,i,j,l: integer;
    c,r,t: integer;
    d: ARRAY [0..m] OF integer;  (*positive powers*)
    f: ARRAY [1..n] OF integer;  (*negative powers*)
BEGIN l := 0; r := 1; d[0] := 1;
  FOR exp := 1 TO n DO
  BEGIN (* compute and print 2**exp *)  c := 0;
    FOR i := 0 TO l DO
    BEGIN t := 2*d[i] + c;
      IF t >= 10 THEN
        BEGIN d[i] := t-10; c := 1
        END
      ELSE
        BEGIN d[i] := t; c := 0
        END
    END ;
    IF c > 0 THEN
      BEGIN l := l+1; d[l] := 1
      END ;
    FOR i := m DOWNTO l DO write(" ");
    FOR i := l DOWNTO 0 DO write(d[i]:1);
    write(exp:5, "  .");
    (*compute and print 2**(-exp) *)
    FOR j := 1 TO exp-1 DO
    BEGIN r := 10*r + f[j];
      f[j] := r DIV 2; r := r - 2*f[j]; write(f[j]:1)
    END ;
    f[exp] := 5; writeln("5"); r := 0
  END
END .
```

4. Compute a table of exact fractions 1/2, 1/3, ... , 1/64. If the fraction has a period, print an apostrophe in front of its first digit and truncate after its last digit.

```
PROGRAM fractions(output);
(* fractions to the base b *)
  CONST b = 10; max = 64;
  VAR i,l,n,q,r: integer;
      a, f: ARRAY [0..max] OF integer;
BEGIN FOR n := 2 TO max DO
  BEGIN l := 0; r := 1;
    FOR i := 0 TO n-1 DO a[i] := 0;
    REPEAT l := l+1; a[r] := l;
      r := b*r; q := r DIV n; r := r - q*n; f[l] := q;
    UNTIL a[r] # 0;
    write(n, " ", ".");
    FOR i := 1 TO a[r]-1 DO  write(f[i]:1);
    IF a[r] > 1 THEN write("'");
    FOR i := a[r] TO l DO  write(f[i]:1);
    writeln
  END
END .
```

5. Compute the harmonic function H(n) = 1 + 1/2 + 1/3 + ... + 1/n with m digits accuracy.

```
PROGRAM harmonic(input,output);
  CONST lim = 100;
  VAR i,k,m,n,c,r,q,sum: integer;
      d,s: ARRAY [0..lim] OF integer;
BEGIN read(m,n);
  IF (m>0) AND (m<lim) THEN
  BEGIN d[0] := 0; s[0] := 1;
    FOR i := 1 TO m DO s[i] := 0;
    FOR k := 2 TO n DO
    BEGIN (*compute 1/k)* r := 1;
      FOR i := 1 TO m DO
        BEGIN r := 10*r; q := r DIV k; r := r-q*k; d[i] := q
        END ;
      IF (10*r DIV k) >= 5 THEN d[m] := d[m]+1;  (*round)*
      write(" 0."); (*intermediate output*)
      FOR i := 1 TO m DO write(d[i]:1);
      writeln;
      (*compute s := s + 1/k)*  c := 0;
      FOR i := m DOWNTO 0 DO
        BEGIN sum := s[i]+d[i]+c;
          IF sum >= 10 THEN
            BEGIN sum := sum-10; c := 1
            END
          ELSE c := 0;
          s[i] := sum
        END
    END ;
    write(" ", s[0]:1, ".");
    FOR i := 1 TO m DO write(s[i]:1);
    writeln
  END
END .
```

6. Compute a table of the first n prime numbers. Print m numbers per line.

```
PROGRAM primes(output);
CONST n = 1000; n1 = 33; m = 20;   (*n1 ~ sqrt(n)*)
VAR i,k,x,inc,lim,square,l: integer;
    prim: boolean;
    p,v: ARRAY [1..n1] OF integer;
BEGIN l := 0;
   x := 1; inc := 4; lim := 1; square := 9;
   FOR i := 3 TO n DO
   BEGIN (*find next prime*)
      REPEAT x := x+inc; inc := 6-inc;
         IF square <= x THEN
            BEGIN lim := lim+1;
               v[lim] := square; square := sqr(p[lim+1])
            END ;
         k := 2; prim := true;
         WHILE prim AND (k<lim) DO
         BEGIN k := k+1;
            IF v[k] < x THEN v[k] := v[k] + 2*p[k];
            prim := x # v[k]
         END
      UNTIL prim;
      IF i <= n1 THEN p[i] := x;
      write(x:6); l := l+1;
      IF l = m THEN
         BEGIN writeln; l := 0
         END
   END ;
   writeln
END .
```

7. Compute a table of the first n prime numbers. Print m numbers per line. Use the method of the sieve of Eratosthenes.

```
PROGRAM primes(output);
CONST m = 100; n = 10000; m = 20; h = 58;
VAR x, inc, i,k, x1,x2, lim, square, a,b,l: integer;
    p,v: ARRAY [1..m] OF integer;
    sieve: SET OF 0..h;
BEGIN l := 0;
   x := 1; inc := 4; lim := 1; square := 9;
   x1 := 0; x2 := 0; sieve := [0..h];
   p[1] := 2; p[2] := 3;
   FOR i := 3 TO n DO
   BEGIN (*find next prime)*
      REPEAT x := x+inc; inc := 6-inc;
         IF x >= square THEN
         BEGIN lim := lim+1; a := square; b := 2*p[lim];
            WHILE a < x2 DO
              BEGIN sieve := sieve - [a-x1]; a := a+b
              END ;
            v[lim] := a; square := sqr(p[lim+1])
         END ;
         IF x >= x2 THEN
         BEGIN (*construct new sieve)*
            x1 := x2; x2 := x2+h; sieve := [0..h];
            FOR k := 3 TO lim DO
            BEGIN a := v[k]; b := 2*p[k];
               WHILE a < x2 DO
                 BEGIN sieve := sieve - [a-x1]; a := a+b
                 END ;
               v[k] := a
            END
         END
      UNTIL x-x1 IN sieve;
      IF i <= m THEN p[i] := x;
      write(x:6); l := l+1;
      IF l = m THEN
        BEGIN writeln; l := 0
        END
   END ;
   writeln
END .
```

3. Real (floating point) arithmetic

1. Compute the sum 1 - 1/2 + 1/3 - 1/4 + ... - 1/10000 in four different ways:
   1. proceed strictly from left to right,
   2. sum positive and negative terms separately,
   3. proceed strictly from right to left,
   4. as in 2., but from right to left.

Explain the differences in the results.

```
PROGRAM sum10000(output);
   CONST n = 10000;
   VAR i: integer;  x, y, s1, s2p, s2n: real;
BEGIN i := 1;
   s1 := 0; s2p := 0; s2n := 0;
   REPEAT x := 1.0/i; y := 1.0/(i+1);
      s1 := s1 + x - y;
      s2p := s2p + x; s2n := s2n + y;
      i := i+2
   UNTIL i > n;
   write (s1, s2p-s2n);
   i := n;
   s1 := 0; s2p := 0; s2n := 0;
   REPEAT x := 1.0/(i-1); y := 1.0/i;
      s1 := s1 + x - y;
      s2p := s2p + x; s2n := s2n + y;
      i := i-2
   UNTIL i = 0;
   writeln(s1, s2p-s2n)
END .
```

2. Multiply the complex number z = 5/13 + 12/13i 50 times with the complex number w = (0.6 + 0.8i). Print intermediate products and the square of their absolute value. Note that |z| = |w| = 1.

```
PROGRAM complexmull(output);
   CONST u = 0.6; v = 0.8;
   VAR i,j: integer;  x,x1,y: real;
BEGIN x := 5/13; y := 12/13;
   FOR i := 1 TO 50 DO
   BEGIN FOR j := 1 TO 10 DO
         BEGIN (* (x+iy) := (x+iy) * (u+iv)  *)
            x1 := x*u - y*v; y := y*u + x*v; x := x1
         END ;
         writeln(x,y,sqr(x)+sqr(y))
   END
END .
```

3. Compute the Fibonacci numbers F(1) ... F(N) in two different ways:
   1. By repeated addition according to F(n) = F(n-1) + F(n-2), F(0) = F(1) = 1,
   2. Using the formula F(n) ~ (phi↑n)/sqrt(5), where phi = (1+sqrt(5))/2.

Terminate as soon as the two results differ.

```
PROGRAM fibonacci(output);
  CONST root5 = 2.236068;
  VAR i, fib0, fib1, fib3, t: integer;
    phi,fib2: real;
BEGIN phi := (1.0+root5)/2;
  i := 0; fib0 := 1; fib1 := 0; fib2 := 1.0 / root5;
  REPEAT i := i+1;
    t := fib0+fib1; fib0 := fib1; fib1 := t;
    fib2 := fib2 * phi; fib3 := trunc(fib2 + 0.5);
    writeln(i, fib1, fib3)
  UNTIL fib1 # fib3
END .
```

4. Analytic functions and iteration

```
CONST eps = 1E-8;

FUNCTION sqrt (x:real): real;
   VAR a,c: real;          (* 0<x<2 *)
BEGIN a := x; c := 1.0-x;     (* abs(c)<1 *)
   REPEAT (* a↑2 = b*(1-c) =>  (a*(1+c/2))↑2 = b*(1-c)*(1+c/2)↑2 =
          b*(1-0.75*(c↑2) - 0.25*(c↑3))  *)
      a := a*(1.0 + 0.5*c);
           (* a↑2 = b*(1-0.75*(c↑2) - 0.25*(c↑3)) =
          = b*(1-(c↑2)*(0.75 + 0.25*c)  *)
      c := sqr(c) * (0.75 + 0.25*c);
           (* a↑2 = b*(1-c)  *)
   UNTIL abs(c) < eps;
   sqrt := a
END ;

FUNCTION log (x: real): real;
   VAR a,b,s: real;          (* 1<=x<2 *)
BEGIN a := x; b := 1.0; s := 0;
   REPEAT (* log(x) = s + b*log(a), b<=1, 1<=a<2 *)
      a := sqr(a);   (* log(x) = s + b*log(sqrt(a)), 1<=a<4 *)
      b := 0.5*b;    (* log(x) = s + b*log(a) *)
      IF a >= 2.0 THEN
      BEGIN (* 2<a<4 *) s := s+b  (* log(x) = s + (1-b)*log(a) *);
         a := 0.5*a
      END
   UNTIL abs(b) < eps;
   log := s
END ;

FUNCTION recip (x: real): real;
   VAR a,c: real;          (* 0<x<2 *)
BEGIN a := 1.0; c := 1.0 - x;
   REPEAT (* a*x = 1-c, abs(c)<1 *)
      a := a*(1.0+c);  (* x*a = (1-c)*(1+c) = 1 - c↑2 *)
      c := sqr(c);     (* x*a = 1-c *)
   UNTIL abs(c) < eps;
   recip := a  (* recip = 1/x *)
END ;
```

Compute analytic functions as truncated sums. Determine the recurrence relations of their terms.

```
PROGRAM recurrence(input,output);
  VAR i,n: integer; x,y,s,t: real;
BEGIN
  writeln(" exp"); n := 5;
  REPEAT read(x); y := 1.0; i := 0; t := 1.0;
    REPEAT i := i+1; t := t*x/i;
      y := y + t
    UNTIL y+t = y;
    writeln(x,y,i);  n := n-1
  UNTIL n = 0;

  writeln(" sin"); n := 5;
  REPEAT read(x); y := x; i := 1; s := sqr(x); t := x;
    REPEAT i := i+2; t := -t*s/((i-1)*i);
      y := y + t
    UNTIL y+t = y;
    writeln(x,y, i DIV 2);  n := n-1
  UNTIL n = 0;

  writeln(" cos"); n := 5;
  REPEAT read(x); y := 1.0; i := 0; s := sqr(x); t := 1.0;
    REPEAT i := i+2; t := -t*s/((i-1)*i);
      y := y + t
    UNTIL y+t = y;
    writeln(x,y, i DIV 2);  n := n-1
  UNTIL n = 0;

  writeln(" arcsin"); n := 5;
  REPEAT read(x); y := x; i := 1; s := sqr(x); t := x;
    REPEAT i := i+2; t := t*s*sqr(i-2)/((i-1)*i);
      y := y + t
    UNTIL y+t = y;
    writeln(x,y, i DIV 2);  n := n-1
  UNTIL n = 0;

  writeln(" arctan"); n := 5;
  REPEAT read(x); y := x; i := 1; s := sqr(x); t := x;
    REPEAT i := i+2; t := -t*s*(i-2)/i;
      y := y + t
    UNTIL y+t = y;
    writeln(x,y, i DIV 2);  n := n-1
  UNTIL n = 0;

  writeln(" ln"); n := 5;
  REPEAT read(x); x := x-1.0; y := x; t := x; i := 1;
    REPEAT i := i+1; t := -t*x*(i-1)/i;
      y := y + t
    UNTIL y+t = y;
    writeln(x+1.0, y, i);  n := n-1
  UNTIL n = 0;
END .
```

5. Text processing

1. Plot the function f(x) = exp(-x) * cos(2*pi*x) with your line printer in the range x = 0 ... 4. Use 32 lines for the unit coordinate.

```
PROGRAM printerplot(output);
  CONST xscale = 32;
    yscale = 50; yshift = 65;
    twopi = 6.2831833071796;
  VAR i,k,n: integer;
    x,y: real;
BEGIN n := 0;  (* n = x position *)
  REPEAT x := n / xscale;
    y := exp(-x)*cos(x*twopi); k := round(y*yscale);
    i := 0; write(" ");  (* i = no of chars in line *)
    IF k < 0 THEN
    BEGIN write(" ": yshift+k); write("*");
      k := -k-1; IF k > 0 THEN write(" ":k);
      write("|")
    END ELSE
    BEGIN write(" ": yshift);
      IF k > 0 THEN
      BEGIN write("|"); k := k-1;
        IF k > 0 THEN write(" ":k)
      END ;
      write("*")
    END ;
    writeln; n := n+1
  UNTIL n > 96
END .
```

2. Read a text and count the number of words with length 1, 2, ... , 20, and those with length greater than 20. Words are separated by blanks or ends of lines.

```
PROGRAM wordlengths(input,output);
  VAR i,k: integer;
      ch: char;
      count: ARRAY [1..21] OF integer;
BEGIN
  FOR i := 1 TO 21 DO count[i] := 0;
  WHILE NOT eof(input) DO
  BEGIN read(ch);
    IF ("a"<=ch) AND (ch<="z") THEN
    BEGIN (*new word)* k := 0;
      REPEAT k := k+1; read(ch)
      UNTIL (ch<"a") OR ("z"<ch) ;
      IF k > 20 THEN k := 21;
      count[k] := count[k] + 1
    END
  END ;
  writeln;
  writeln("   length     count");
  FOR i := 1 TO 21 DO writeln(i,count[i])
END .
```

3. Read a text and produce a copy with flushed left and right margins. Place a fixed number of characters (say, length = 72) in each line, and distribute blanks as word separators accordingly.

```
PROGRAM edit(input,output);
  CONST length = 72;
  VAR ch: char;
    i,m,k,lim: integer;
    line:  ARRAY [1..136] OF char;
    index: ARRAY [1.. 68] OF integer;

  PROCEDURE setline;
    VAR i,j,h,s: integer;
        spaces, q,l,r: integer;
  BEGIN IF m=0 THEN
      BEGIN (*word is longer than line)* m := 1; index[m] := lim
      END ;
    j := 0; write(" "); (*printer control)*
    IF m > 1 THEN
    BEGIN spaces := lim - index[m];
      q := spaces DIV (m-1); r := spaces - (m-1)*q;
      l := (m-r) DIV 2; r := l+r;   (*distribute spaces)*
      i := 0;
      REPEAT i := i+1; s := index[i];
        REPEAT j := j+1; write(line[j])
        UNTIL j = s;
        FOR h := 1 TO q DO write(" ");
        IF (l<=i) AND (i<r) THEN write(" ");
      UNTIL i = m-1
    END ;
    s := index[m] -1;
    REPEAT j := j+1; write(line[j])
    UNTIL j = s;
    j := 0; writeln;
    FOR h := index[m]+1 TO lim DO
      BEGIN j := j+1; line[j] := line[h]
      END ;
    k := j; m := 0
  END (*setline)* ;

BEGIN lim := length+1;
  k := 0;   (*k = no. OF characters IN line)*
  m := 0;   (*m = no. OF complete words IN line)*
  WHILE NOT eof(input) DO
  BEGIN read(ch);
    IF ch # " " THEN
    BEGIN (*next word)*
      REPEAT k := k+1; line[k] := ch; read(ch);
        IF k = lim THEN setline
      UNTIL ch = " ";
      k := k+1; line[k] := " ";
      m := m+1; index[m] := k;
      IF k = lim THEN setline
    END
  END ;
  write(" ");
  FOR i := 1 TO k DO write(line[i]);
  writeln
```

```
END .
```

4. Read a text and replace any sequence of one or more blanks by a single blank.

```
PROGRAM crunch(input,output);
   CONST blank = " ";
   VAR ch: char;
BEGIN
  WHILE NOT eof(input) DO
  BEGIN read(ch); write(blank);  (*printer control*)
    WHILE ch = blank DO read(ch);
    WHILE NOT eoln(input) DO
    BEGIN
      REPEAT write(ch); read(ch)
      UNTIL ch = blank;
      write(blank);
      WHILE (ch=blank) AND NOT eoln(input) DO read(ch)
    END ;
    writeln; read(ch)
  END
END .
```

5. A record company conducts a poll to evaluate its products. The most popular hits are to be broadcast in a hit parade. The polled population is divided into four categories according to sex and age (teenager <= 20, adult > 20). Each person is asked to list five hits, identified by their number between 1 and, say, 50. The result of the poll is presented as a file; each record lists a respondent's name, first name, sex, age, and his choices ordered according to priority. A program is to compute the following data:

1. A list of hits ordered according to popularity. Each entry consists of the hit number and the number of votes it received. Hits not mentioned are omitted.

2. Four separate lists with names and first names of all respondents who had mentioned in first place one of the three hits most popular in their category.

```
PROGRAM hitparade(poll ,output);
  CONST n = 50;   (* number of hits *)
  TYPE sex = (male, female);
     hitno = 1 .. n;
     query = RECORD
               name, firstname: alfa;
               s: sex;
               age: 0 .. 99;
               choice: PACKED ARRAY [1..5] OF hitno
             END ;
  VAR i,k,max: integer;
      b: boolean;
      total: ARRAY [hitno] OF integer;
      count: ARRAY [sex,boolean,hitno] OF integer;
      poll:  FILE OF query;

  PROCEDURE findnames(x: sex; y: boolean);
    VAR i,j,k: integer;
        selection: SET OF hitno;
  BEGIN selection := []; reset(poll);
    writeln(" ----------------------------------------");
    (* find 3 hits most frequently listed in this group *)
    FOR i := 1 TO 3 DO
    BEGIN max := 0;
      FOR j := 1 TO n DO
        IF max < count[x,y,j] THEN
          BEGIN max := count[x,y,j]; k := j
          END ;
      selection := selection + [k]; count[x,y,k] := 0
    END ;
    (* list persons with one of these hits as first choice *)
    WHILE NOT eof(poll) DO
    BEGIN
      WITH poll↑ DO
      IF s = x THEN
      IF (age >= 20) = y THEN
      IF choice[1] IN selection THEN
         writeln("  ",name,"  ",firstname);
      get(poll)
    END
  END (*findnames*) ;

BEGIN reset(poll);
  FOR i := 1 TO n DO
    BEGIN total[i] := 0;
```

```
      count[male,true,i] := 0; count[female,true,i] := 0;
      count[male,false,i] := 0; count[female,false,i] := 0
    END ;
  (* collect counts *)
  WHILE NOT eof(poll) DO
  BEGIN
    WITH poll↑ DO
    FOR i := 1 TO 5 DO
      BEGIN b := age >= 20; k := choice[i];
        count[s,b,k] := count[s,b,k] + 1
      END ;
    get(poll)
  END ;
  (* compute totals *)
  FOR i := 1 TO n DO
    total[i] := count[male,true,i] + count[female,true,i]
              + count[male,false,i] + count[female,false,i];
  page(output);
  writeln(' report on hit popularity poll');
  writeln(' list of hits ordered after popularity');
  writeln('       hit     frequency');
  REPEAT max := 0; k := 0;
    FOR i := 1 TO n DO
      IF max < total[i] THEN
        BEGIN max := total[i]; k := i
        END ;
    IF max > 0 THEN
      BEGIN total[k] := 0; writeln(k, max)
      END ;
  UNTIL max = 0;
  writeln(' namelists separate by sex and age');
  writeln(' men   '); findnames(male,true);
  writeln(' women'); findnames(female,true);
  writeln(' boys '); findnames(male,false);
  writeln(' girls'); findnames(female,false);
  writeln(' end of report')
END .
```

6. Recursion

1. Compute all n! permutations of the integers 1 ... n.

```
PROGRAM permute(output);
  CONST n = 4;
  VAR i: integer;
      a: ARRAY [1..n] OF integer;

  PROCEDURE print;
    VAR i: integer;
  BEGIN FOR i := 1 TO n DO write(a[i]:3);
    writeln
  END (*print*) ;

  PROCEDURE perm(k: integer);
    VAR i,x: integer;
  BEGIN
    IF k = 1 THEN print ELSE
    BEGIN perm(k-1);
      FOR i := 1 TO k-1 DO
      BEGIN x := a[i]; a[i] := a[k]; a[k] := x;
        perm(k-1);
          x := a[i]; a[i] := a[k]; a[k] := x;
      END
    END
  END (*perm*) ;

BEGIN
  FOR i := 1 TO n DO a[i] := i;
  perm(n)
END .
```

2. Convert expressions from infix to postfix form. Each expression is written on a separate line. Expressions have the following syntax:

```
expression =    term {("+"|"=") term}.
term =          factor {"*" factor}.
factor =        letter | "(" expression ")".
```

```
PROGRAM postfix(input,output);
  VAR ch: char;
  PROCEDURE expression;
    VAR op: char;
    PROCEDURE factor;
    BEGIN IF ch = "(" THEN
          BEGIN read(ch); expression; read(ch)  (* )  *)
          END ELSE
          BEGIN write(ch); read(ch)
          END
    END (* factor *) ;
    PROCEDURE term;
    BEGIN factor;
          WHILE ch = "*" DO
          BEGIN read(ch); factor; write("*")
          END
    END (* term *) ;
  BEGIN term;
        WHILE (ch="+") OR (ch="-") DO
        BEGIN op := ch; read(ch); term; write(op)
        END
  END (* expression *) ;
BEGIN
  WHILE NOT eof(input) DO
  BEGIN write(" "); read(ch); expression; writeln; readln
  END
END .
```

3. Plot Hilbert curves of orders 1 ... n. Plot procedure produces output for the Tektronix 4010 terminal. Data are represented as 12-bit bytes: a call of procedure p12 appends a byte to the output file.

```
PROGRAM hilbert(pf,output);
CONST n = 4; h0 = 512;
VAR i,h,x,y,x0,y0: integer;
   cc, wc, buf: integer;
   pf: FILE OF integer;     (*plot file*)

PROCEDURE p12(u: integer);
BEGIN buf := buf * 4096 + u;  cc := cc + 1;
   IF cc = 5 THEN
   BEGIN pf↑ := buf; put(pf);
      wc := wc+1; buf := 0; cc := 0;
      IF wc = 31 THEN
         BEGIN pf↑ := 0; put(pf); wc := 0
         END
   END
END (*p12*) ;

PROCEDURE plot;
   VAR u,v: integer;
BEGIN u := x DIV 32; v := y DIV 32;
   p12(40b+v); p12(140b+y-32*v);
   p12(40b+u); p12(100b+x-32*u);
END (*plot*) ;

PROCEDURE setplot;
BEGIN p12(35b); plot
END ;

PROCEDURE startplot;
BEGIN cc := 0; wc := 0; buf := 0; rewrite(pf)
END ;

PROCEDURE endplot;
BEGIN x := 0; y := 767; setplot; p12(37b);
   REPEAT p12(0) UNTIL cc = 0
END ;

PROCEDURE b(i: integer); FORWARD;
PROCEDURE c(i: integer); FORWARD;
PROCEDURE d(i: integer); FORWARD;

PROCEDURE a(i: integer);
BEGIN IF i > 0 THEN
      BEGIN d(i-1); x := x-h; plot;
            a(i-1); y := y-h; plot;
            a(i-1); x := x+h; plot;
            b(i-1)
      END
END ;

PROCEDURE b;
BEGIN IF i > 0 THEN
      BEGIN c(i-1); y := y+h; plot;
            b(i-1); x := x+h; plot;
```

```
            b(i-1); y := y-h; plot;
            a(i-1)
      END
END ;

PROCEDURE c;
BEGIN IF i > 0 THEN
      BEGIN b(i-1); x := x+h; plot;
            c(i-1); y := y+h; plot;
            c(i-1); x := x-h; plot;
            d(i-1)
      END
END ;

PROCEDURE d;
BEGIN IF i > 0 THEN
      BEGIN a(i-1); y := y-h; plot;
            d(i-1); x := x-h; plot;
            d(i-1); y := y+h; plot;
            c(i-1)
      END
END ;

BEGIN startplot;
   i := 0; h := h0; x0 := h DIV 2 + h; y0 := h DIV 2;
   REPEAT (*plot hilbert curve OF order i*)
      i := i+1; h := h DIV 2;
      x0 := x0 + (h DIV 2); y0 := y0 + (h DIV 2);
      x := x0; y := y0; setplot;
      a(i)
   UNTIL i = n;
   endplot
END .
```

4. Plot Sierpinski space-filling curves using their recursive pattern. Plot routine is identical to the one used in the preceding program.

```
PROGRAM sierpinski(pf,output);
CONST n = 4; h0 = 512;
VAR i,h,x,y,x0,y0: integer;
   cc, wc, buf: integer;
   pf: FILE OF integer;     (*plot file*)

PROCEDURE p12(u: integer);
BEGIN buf := buf * 4096 + u;  cc := cc + 1;
   IF cc = 5 THEN
   BEGIN pf↑ := buf; put(pf);
      wc := wc+1; buf := 0; cc := 0;
      IF wc = 31 THEN
         BEGIN pf↑ := 0; put(pf); wc := 0
         END
   END
END (*p12*) ;

PROCEDURE plot;
   VAR u,v: integer;
BEGIN u := x DIV 32; v := y DIV 32;
   p12(40b+v); p12(140b+y-32*v);
   p12(40b+u); p12(100b+x-32*u);
END (*plot*) ;

PROCEDURE setplot;
BEGIN p12(35b); plot
END ;

PROCEDURE startplot;
BEGIN cc := 0; wc := 0; buf := 0; rewrite(pf)
END ;

PROCEDURE endplot;
BEGIN x := 0; y := 767; setplot; p12(37b);
   REPEAT p12(0) UNTIL cc = 0
END ;

PROCEDURE b(i: integer); FORWARD;
PROCEDURE c(i: integer); FORWARD;
PROCEDURE d(i: integer); FORWARD;


PROCEDURE a(i: integer);
BEGIN IF i > 0 THEN
      BEGIN a(i-1); x := x+h; y := y-h; plot;
            b(i-1); x := x + 2*h; plot;
            d(i-1); x := x+h; y := y+h; plot;
            a(i-1)
      END
END ;

PROCEDURE b;
BEGIN IF i > 0 THEN
      BEGIN b(i-1); x := x-h; y := y-h; plot;
            c(i-1); y := y - 2*h; plot;
```

```
            a(i-1); x := x+h; y := y-h; plot;
            b(i-1)
      END
END ;

PROCEDURE c;
BEGIN IF i > 0 THEN
      BEGIN c(i-1); x := x-h; y := y+h; plot;
            d(i-1); x := x - 2*h; plot;
            b(i-1); x := x-h; y := y-h; plot;
            c(i-1)
      END
END ;

PROCEDURE d;
BEGIN IF i > 0 THEN
      BEGIN d(i-1); x := x+h; y := y+h; plot;
            a(i-1); y := y + 2*h; plot;
            c(i-1); x := x-h; y := y+h; plot;
            d(i-1)
      END
END ;

BEGIN startplot;
   i := 0; h := h0 DIV 4; x0 := 2*h; y0 := 3*h;
   REPEAT i := i+1; x0 := x0-h;
      h := h DIV 2; y0 := y0+h;
      x := x0; y := y0; setplot;
      a(i); x := x+h; y := y-h; plot;
      b(i); x := x-h; y := y-h; plot;
      c(i); x := x-h; y := y+h; plot;
      d(i); x := x+h; y := y+h; plot;
   UNTIL i = n;
   endplot
END .
```

7. Sorting arrays

Reference: N.Wirth, *Algorithms + Data Structures = Programs*, Prentice-Hall, Inc., 1975

```
PROGRAM sort(output);
  CONST n = 256; nn = 512;
  TYPE index = 0..nn;
    item = RECORD key: integer;
             (*other fields defined here*)
           END ;

  VAR i : index;  r: integer;
    a: ARRAY [-15..nn] OF item;
    z: ARRAY [1..n] OF integer;

PROCEDURE test(t: alfa; PROCEDURE sort);
  VAR i,z: integer;
BEGIN write(" ", t);
  FOR i := 1 TO n DO a[i].key := i;
  z := clock; sort; write(clock-z);
  FOR i := 1 TO n DO a[i].key := z[i];
  z := clock; sort; write(clock-z);
  FOR i := 1 TO n DO a[i].key := n-i;
  z := clock; sort; writeln(clock-z);
END (*test*) ;

PROCEDURE straightinsertion;
  VAR i,j: index;  x: item;
BEGIN
  FOR i := 2 TO n DO
  BEGIN x := a[i]; a[0] := x; j := i-1;
    WHILE x.key < a[j].key DO
      BEGIN a[j+1] := a[j]; j := j-1;
      END ;
    a[j+1] := x
  END
END ;

PROCEDURE binaryinsertion;
  VAR i,j,l,r,m: index;  x: item;
BEGIN
  FOR i := 2 TO n DO
  BEGIN x := a[i]; l := 1; r := i-1;
    WHILE l <= r DO
    BEGIN m := (l+r) DIV 2;
      IF x.key < a[m].key THEN r := m-1 ELSE l := m+1;
    END ;
    FOR j := i-1 DOWNTO l DO a[j+1] := a[j];
    a[l] := x;
  END
END ;
```

```
PROCEDURE shellsort;
   CONST t = 4;
   VAR i,j,k,s: index;  x: item;  m: 1..t;
      h: ARRAY [1..t] OF integer;
BEGIN h[1] := 9; h[2] := 5; h[3] := 3; h[4] := 1;
   FOR m := 1 TO t DO
BEGIN k := h[m]; s := -k;  (*sentinel position*)
      FOR i := k+1 TO n DO
      BEGIN x := a[i]; j := i-k;
         IF s=0 THEN s := -k; s := s+1; a[s] := x;
         WHILE x.key < a[j].key DO
         BEGIN a[j+k] := a[j];  j := j-k
         END ;
         a[j+k] := x
      END
   END ;
END  ;

PROCEDURE straightselection;
   VAR i,j,k: index;  x: item;
BEGIN FOR i := 1 TO n-1 DO
      BEGIN k := i;  x := a[i];
         FOR j := i+1 TO n DO
            IF a[j].key < x.key THEN
            BEGIN k := j; x := a[j]
            END ;
         a[k] := a[i]; a[i] := x;
      END
END ;

PROCEDURE heapsort;
   VAR l,r: index;  x: item;

   PROCEDURE sift;
      LABEL 13;
      VAR i,j: index;
   BEGIN i := l; j := 2*i; x := a[i];
      WHILE j <= r DO
      BEGIN IF j < r THEN
               IF a[j].key < a[j+1].key THEN j := j+1;
         IF x.key >= a[j].key THEN GOTO 13;
         a[i] := a[j]; i := j; j := 2*i
      END ;
   13: a[i] := x
   END ;

BEGIN l := (n DIV 2) + 1; r := n;
   WHILE l > 1 DO
      BEGIN l := l-1; sift
      END ;
   WHILE r > 1 DO
      BEGIN x := a[1]; a[1] := a[r]; a[r] := x;
         r := r-1; sift
      END
END (*heapsort*) ;
```

```
PROCEDURE bubblesort;
   VAR i,j: index;  x: item;
BEGIN FOR i := 2 TO n DO
      BEGIN FOR j := n DOWNTO i DO
            IF a[j-1].key > a[j].key THEN
            BEGIN x := a[j-1]; a[j-1] := a[j]; a[j] := x;
            END ;
      END
END (*bubblesort*) ;

PROCEDURE bubblex;
   VAR j,k,l: index;  x: item;
BEGIN l := 2;
   REPEAT k := n;
      FOR j := n DOWNTO l DO
      IF a[j-1].key > a[j].key THEN
         BEGIN x := a[j-1]; a[j-1] := a[j]; a[j] := x;
            k := j
         END ;
      l := k+1
   UNTIL l > n
END (*bubblex*) ;

PROCEDURE shakersort;
   VAR j,k,l,r: index;  x:item;
BEGIN l := 2; r := n; k := n;
   REPEAT
      FOR j := r DOWNTO l DO
         IF a[j-1].key > a[j].key THEN
         BEGIN x := a[j-1]; a[j-1] := a[j]; a[j] := x;
            k := j
         END ;
      l := k+1;
      FOR j := l TO r DO
         IF a[j-1].key > a[j].key THEN
         BEGIN x := a[j-1]; a[j-1] := a[j]; a[j] := x;
            k := j
         END ;
      r := k-1;
   UNTIL l > r
END (*shakersort*) ;
```

```
PROCEDURE quicksort;   (*recursive*)

   PROCEDURE sort(l,r: index);
      VAR i,j: index; x,w: item;
   BEGIN i := l; j := r;
      x := a[(l+r) DIV 2];
      REPEAT
         WHILE a[i].key < x.key DO i := i+1;
         WHILE x.key < a[j].key DO j := j-1;
         IF i <= j THEN
         BEGIN w := a[i]; a[i] := a[j]; a[j] := w;
            i := i+1; j := j-1
         END
      UNTIL i > j;
      IF l < j THEN sort(l,j);
      IF i < r THEN sort(i,r)
   END ;

BEGIN sort(1,n)
END (*quicksort*) ;

PROCEDURE quicksort1;   (*non-recursive*)
   CONST m = 12;
   VAR i,j,l,r: index;
      x,w: item;
      s: 0 .. m;
      stack: ARRAY [1..m] OF
            RECORD l,r: index END ;
BEGIN s := 1; stack[1].l := 1; stack[1].r := n;
   REPEAT (*take top request from stack*)
      l := stack[s].l; r := stack[s].r; s := s-1;
      REPEAT (*split a[l] ... a[r]*)
         i := l; j := r; x := a[(l+r) DIV 2];
         REPEAT
            WHILE a[i].key < x.key DO i := i+1;
            WHILE x.key < a[j].key DO j := j-1;
            IF i <= j THEN
            BEGIN w := a[i]; a[i] := a[j]; a[j] := w;
               i := i+1; j := j-1
            END
         UNTIL i > j;
         IF i < r THEN
         BEGIN (*stack request to sort right partition*)
            s := s+1; stack[s].l := i; stack[s].r := r
         END ;
         r := j
      UNTIL l >= r
   UNTIL s = 0
END (*quicksort1*) ;
```

```
PROCEDURE mergesort;
   VAR i,j,k,l,t: index;
      h,m,p,q,r: integer;  up: boolean;
   (*note that a has indices 1..2*n*)
BEGIN up := true; p := 1;
   REPEAT h := 1; m := n;
      IF up THEN
      BEGIN i := 1; j := n; k := n+1; l := 2*n
      END ELSE
      BEGIN k := 1; l := n; i := n+1; j := 2*n
      END ;
      REPEAT  (*merge a run from i and j to k*)
         (*q = length of i-run, r = length of j-run*)
         IF m >= p THEN q := p ELSE q := m;  m := m-q;
         IF m >= p THEN r := p ELSE r := m;  m := m-r;
         WHILE (q#0) AND (r#0) DO
         BEGIN (*merge*)
            IF a[i].key < a[j].key THEN
            BEGIN a[k] := a[i]; k := k+h; i := i+1; q := q-1;
            END ELSE
            BEGIN a[k] := a[j]; k := k+h; j := j-1; r := r-1;
            END
         END ;
         IF q = 0 THEN
            BEGIN (*copy tail of j-run*)
               WHILE r # 0 DO
               BEGIN a[k] := a[j]; k := k+h; j := j-1; r := r-1;
               END
            END ELSE
            BEGIN (*r = 0, copy tail of i-run*)
               WHILE q # 0 DO
               BEGIN a[k] := a[i]; k := k+h; i := i+1; q := q-1;
               END
            END ;
         h := -h; t := k; k := l; l := t
      UNTIL m = 0;
      up := NOT up; p := 2*p
   UNTIL p >= n;
   IF NOT up THEN
   FOR i := 1 TO n DO a[i] := a[i+n]
END (*mergesort*) ;

BEGIN i := 0; r := 54321;
   REPEAT i := i+1;
      r := (131071*r) MOD 2147483647; z[i] := r
   UNTIL i = n;
   test("str insert", straightinsertion);
   test("bin insert", binaryinsertion);
   test("shell sort", shellsort);
   test("str select", straightselection);
   test("heapsort  ", heapsort);
   test("bubblesort", bubblesort);
   test("bubblesort", bubblex);
   test("shakersort", shakersort);
   test("quicksort ", quicksort);
   test("quicksort1", quicksort1);
   test("mergesort ", mergesort);
END .
```

8. Sequential sorting

1. Natural merge sort with 3 files (tapes) and 2 phases.

```
PROGRAM mergesort(input,output);
TYPE item = RECORD key: integer
              (*other fields defined here*)
            END ;
   tape = FILE OF item;
VAR c: tape; n: buf: item;

PROCEDURE list(VAR f: tape);
   VAR x: item;
BEGIN reset(f);
   WHILE NOT eof(f) DO
      BEGIN read(f,x); write(output, x.key: 4)
      END ;
   writeln
END (*list*) ;

PROCEDURE naturalmerge;
   VAR l: integer;   (*no. of runs merged*)
       eor: boolean; (*end-of-run indicator*)
       a,b: tape;

   PROCEDURE copy(VAR x,y: tape);
      VAR buf: item;
   BEGIN read(x, buf); write(y,buf);
      IF eof(x) THEN eor := true ELSE eor := buf.key > x↑.key
   END ;

   PROCEDURE copyrun(VAR x,y: tape);
   BEGIN (*copy one run from x to y*)
      REPEAT copy(x,y) UNTIL eor
   END ;

   PROCEDURE distribute;
   BEGIN (*from c to a and b*)
      REPEAT copyrun(c,a);
         IF NOT eof(c) THEN copyrun(c,b)
      UNTIL eof(c)
   END ;

   PROCEDURE mergerun;
   BEGIN (*from a and b to c*)
      REPEAT
         IF a↑.key < b↑.key THEN
         BEGIN copy(a,c);
            IF eor THEN copyrun(b,c)
         END ELSE
         BEGIN copy(b,c);
            IF eor THEN copyrun(a,c)
         END
      UNTIL eor
   END ;

   PROCEDURE merge;
   BEGIN (*from a and b to c*)
```

```
      REPEAT mergerun; l := l+1
      UNTIL eof(a) OR eof(b);
      WHILE NOT eof(a) DO
      BEGIN copyrun(a,c); l := l+1
      END ;
      WHILE NOT eof(b) DO
      BEGIN copyrun(b,c); l := l+1
      END ;
      list(c)
   END ;

BEGIN
   REPEAT rewrite(a); rewrite(b); reset(c);
      distribute;
      reset(a); reset(b); rewrite(c);
      l := 0; merge;
   UNTIL l = 1
END ;

BEGIN (*main program. read input sequence ending with 0*)
   rewrite(c); read(buf.key);
   REPEAT write(c, buf); read(buf.key)
   UNTIL buf.key = 0;
   list(c);
   naturalmerge;
   list(c)
END .
```

2. Sequential sorting by n-way mergesort. In each phase, data are merged from n/2 files and distributed onto the other n/2 files. The program starts with the generation of a single file with random numbers.

```
PROGRAM balancedmerge(output);
CONST n = 6; nh = 3;          (*no. of tapes*)
TYPE item = RECORD
              key: integer
            END ;
  tape = FILE OF item;
  tapeno = 1..n;
VAR leng, rand: integer;      (*used to generate file*)
  eot: boolean;
  buf: item;
  f0: tape;  (*f0 is the input tape with random numbers*)
  f: ARRAY [1..n] OF tape;

PROCEDURE list(VAR f: tape; n: tapeno);
  VAR z: integer;
BEGIN writeln(' tape', n:2); z := 0;
  WHILE NOT eof(f) DO
  BEGIN read(f, buf); write(output, buf.key: 5); z := z+1;
    IF z = 25 THEN
      BEGIN writeln(output); z := 0;
      END ;
  END ;
  IF z # 0 THEN writeln(output); reset(f)
END (*list*) ;

PROCEDURE tapemergesort;
  VAR i,j,mx,tx: tapeno;
    k1,k2,l: integer;
    x, min: integer;
    t, ta: ARRAY [tapeno] OF tapeno;
BEGIN (*distribute initial runs to t[1] ... t[nh]*)
  FOR i := 1 TO nh DO  rewrite(f[i]);
  j := nh; l := 0;
  REPEAT IF j < nh THEN j := j+1 ELSE j := 1;
    (*copy one run from f0 to tape j*)
    l := l+1;
    REPEAT read(f0, buf); write(f[j], buf)
    UNTIL (buf.key > f0↑.key) OR eof(f0)
  UNTIL eof(f0);
  FOR i := 1 TO n DO t[i] := i;
  REPEAT (*merge from t[1] ... t[nh] to t[nh+1] ... t[n]*)
    IF l < nh THEN k1 := l ELSE k1 := nh;
    (*k1 = no. of input tapes in this phase*)
    FOR i := 1 TO k1 DO
      BEGIN reset(f[t[i]]); list(f[t[i]], t[i]); ta[i] := t[i]
      END ;
    l := 0;   (*l = number of runs merged*)
    j := nh+1;    (*j = index of output tape*)
    REPEAT (*merge a run from t[1] ... t[k1] TO t[j]*)
      k2 := k1; l := l+1;  (*k2 = no. of active input tapes*)
      REPEAT (*select minimal element*)
        i := 1; mx := 1; min := f[ta[1]]↑.key;
        WHILE i < k2 DO
        BEGIN i := i+1; x := f[ta[i]]↑.key;
```

```
            IF x < min THEN
            BEGIN min := x;  mx := i
            END
        END ;
        (*ta[mx] has minimal element, move it to t[j]*)
        read(f[ta[mx]], buf); eot := eof(f[ta[mx]]);
        write(f[t[j]], buf);
        IF eot THEN
        BEGIN rewrite(f[ta[mx]]);  (*eliminate tape*)
          ta[mx] := ta[k2]; ta[k2] := ta[k1];
          k1 := k1-1; k2 := k2-1
        END ELSE
        IF buf.key > f[ta[mx]]↑.key THEN
        BEGIN tx := ta[mx]; ta[mx] := ta[k2]; ta[k2] := tx;
          k2 := k2-1
        END
      UNTIL k2 = 0;
      IF j < n THEN j := j+1 ELSE j := nh+1
    UNTIL k1 = 0;
    FOR i := 1 TO nh DO
      BEGIN tx := t[i]; t[i] := t[i+nh]; t[i+nh] := tx
      END
  UNTIL l = 1;
  reset(f[t[1]]); list(f[t[1]], t[1]);   (*sorted output is on t[1]*)
END (*tapemergesort*) ;

BEGIN (*generate random file f0*)
  leng := 200; rand := 7789; rewrite(f0);
  REPEAT rand := (131071*rand) MOD 2147483647;
    buf.key := rand DIV 2147484; write(f0, buf); leng := leng - 1
  UNTIL leng = 0;
  reset(f0); list(f0,1);
  tapemergesort;
END .
```

3. Polyphase sort program. There are n-1 source files for merging and a single output file. The destination of the merged data changes, when a certain number of runs has been distributed. This number is computed according to a Fibonacci distribution.

```
PROGRAM polysort(output);
CONST n = 6;                    (*no. of tapes*)
TYPE item = RECORD
              key: integer
            END ;
  tape = FILE OF item;
  tapeno = 1..n;
VAR leng, rand: integer;        (*used to generate file*)
  eot: boolean;
  buf: item;
  f0: tape;  (*f0 is the input tape with random numbers*)
  f: ARRAY [1..n] OF tape;

PROCEDURE list(VAR f: tape; n: tapeno);
  VAR z: integer;
BEGIN z := 0;
  writeln(" tape", n:2);
  WHILE NOT eof(f) DO
  BEGIN read(f, buf); write(output, buf.key: 5); z := z+1;
    IF z = 25 THEN
      BEGIN writeln(output); z := 0
      END ;
  END ;
  IF z # 0 THEN writeln(output); reset(f)
END (*list*) ;

PROCEDURE polyphasesort;
  VAR i,j,mx,tn: tapeno;
    k, level: integer;
    a, d: ARRAY [tapeno] OF integer;
      (*a[j] = ideal number of runs on tape j*)
      (*d[j] = number of dummy runs on tape j*)
    dn, x, min, z: integer;
    last: ARRAY [tapeno] OF integer;
      (*last[j] = key of tail item on tape j*)
    t,ta: ARRAY [tapeno] OF tapeno;
      (*mappings of tape numbers*)

  PROCEDURE selecttape;
    VAR i: tapeno; z: integer;
  BEGIN
    IF d[j] < d[j+1] THEN j := j+1 ELSE
    BEGIN IF d[j] = 0 THEN
        BEGIN level := level + 1; z := a[1];
          FOR i := 1 TO n-1 DO
          BEGIN d[i] := z + a[i+1] - a[i]; a[i] := z + a[i+1]
          END
        END ;
        j := 1
    END ;
    d[j] := d[j] -1
  END ;
```

```
    PROCEDURE copyrun;
    BEGIN (*copy one run from f0 to tape j*)
      REPEAT read(f0, buf); write(f[j], buf);
      UNTIL eof(f0) OR (buf.key > f0↑.key);
      last[j] := buf.key
    END ;

  BEGIN (*distribute initial runs*)
    FOR i := 1 TO n-1 DO
      BEGIN a[i] := 1; d[i] := 1; rewrite(f[i])
      END ;
    level := 1; j := 1; a[n] := 0; d[n] := 0;
    REPEAT selecttape; copyrun
    UNTIL eof(f0) OR (j=n-1);
    REPEAT selecttape;
      IF last[j] <= f0↑.key THEN
      BEGIN (*continue old run*)
        copyrun;
        IF eof(f0) THEN d[j] := d[j] + 1 ELSE copyrun
      END
      ELSE copyrun
    UNTIL eof(f0);
    FOR i := 1 TO n-1 DO reset(f[i]);

    FOR i := 1 TO n DO t[i] := i;
    REPEAT (*merge from t[1] ... t[n-1] to t[n]*)
      z := a[n-1]; d[n] := 0; rewrite(f[t[n]]);
      writeln(" level", level:4, "  tape", t[n]:4);
      FOR i := 1 TO n DO writeln(t[i], a[i], d[i]);
      REPEAT k := 0;  (*merge one run*)
        FOR i := 1 TO n-1 DO
        IF d[i]  > 0 THEN d[i] := d[i]-1 ELSE
          BEGIN k := k+1; ta[k] := t[i]
          END ;
        IF k = 0 THEN d[n] := d[n] + 1 ELSE
        BEGIN (*merge one real run from t[1] ... t[k]*)
          REPEAT i := 1; mx := 1;
            min := f[ta[1]]↑.key;
            WHILE i < k DO
            BEGIN i := i+1; x := f[ta[i]]↑.key;
              IF x < min THEN
              BEGIN min := x; mx := i
              END
            END ;
            (*ta[mx] contains minimal element, move it to t[n]*)
            read(f[ta[mx]], buf); eot := eof(f[ta[mx]]);
            write(f[t[n]], buf);
            IF (buf.key > f[ta[mx]]↑.key) OR eot THEN
            BEGIN (*drop this tape*)
              ta[mx] := ta[k]; k := k-1
            END
          UNTIL k = 0
        END ;
        z := z-1
      UNTIL z = 0;
      reset(f[t[n]]); list(f[t[n]], t[n]);   (*rotate tapes*)
      tn := t[n]; dn := d[n]; z := a[n-1];
      FOR i := n DOWNTO 2 DO
```

```
        BEGIN t[i] := t[i-1]; d[i] := d[i-1]; a[i] := a[i-1] - z
        END ;
      t[1] := tn; d[1] := dn; a[1] := z;
      (*sorted output is on t[1]*)
      level := level - 1
    UNTIL level = 0;
END (*polyphasesort*) ;

BEGIN (*generate random file*)
    leng := 200; rand := 7789;
    REPEAT rand := (131071*rand) MOD 2147483647;
      buf.key := rand DIV 2147484; write(f0, buf); leng := leng - 1
    UNTIL leng = 0;
    reset(f0); list(f0,1);
    polyphasesort;
END .
```

9. "Problem solving", backtracking.

1. Find all settings of 8 queens on an 8x8 chess board such that no queen checks another queen. [see also, Comm. ACM 14, 4, 221-27 (April 74)].

```
PROGRAM eightqueens(output);
VAR i : integer;
    a : ARRAY [ 1..8 ] OF boolean;
    b : ARRAY [ 2..16] OF boolean;
    c : ARRAY [-7..7 ] OF boolean;
    x : ARRAY [ 1..8 ] OF integer;
    safe : boolean;

  PROCEDURE print;
    VAR k: integer;
  BEGIN write(" ");
    FOR k := 1 TO 8 DO write(x[k]:2);
    writeln
  END ;

PROCEDURE trycol(j : integer);
  VAR i : integer;

  PROCEDURE setqueen;
  BEGIN a[i] := false; b[i+j] := false; c[i-j] := false
  END ;

  PROCEDURE removequeen;
  BEGIN a[i] := true; b[i+j] := true; c[i-j] := true
  END ;
BEGIN i := 0;
    REPEAT i := i+1; safe := a[i] AND b[i+j] AND c[i-j];
      IF safe THEN
      BEGIN setqueen; x[j] := i;
        IF j < 8 THEN trycol(j+1) ELSE print;
        removequeen
      END
    UNTIL i = 8
END;

BEGIN FOR i := 1 TO 8 DO a[i] := true;
    FOR i := 2 TO 16 DO b[i] := true;
    FOR i := -7 TO 7 DO c[i] := true;
    trycol(1);
END.
```

2. Find sequences of digits 0, 1, 2 and of lengths 1 ... 90, such that they contain no two adjacent subsequences that are equal.

```
PROGRAM sequence012(output);
  CONST maxlength = 90;
  VAR n: integer;
    good: boolean;
    s: ARRAY [1..maxlength] OF integer;

  PROCEDURE printsequence;
    VAR k: integer;
  BEGIN write(" ");
    FOR k := 1 TO n DO write(s[k]:1);
    writeln
  END  (*printsequence*) ;

  PROCEDURE changesequence;
  BEGIN IF s[n] = 3 THEN
        BEGIN n := n-1; changesequence
        END ELSE s[n] := succ(s[n])
  END (*changesequence*) ;

  PROCEDURE try;
    VAR i,l,nhalf: integer;
  BEGIN IF n <= 1 THEN good := true ELSE
        BEGIN l := 0; nhalf := n DIV 2;
          REPEAT l := l+1; i := 0;
            (* compare tails of length l for equality *)
            REPEAT good := s[n-i] # s[n-l-i]; i := i+1
            UNTIL good OR (i=l)
          UNTIL NOT good OR (l>=nhalf);
        END
  END (*try*) ;

BEGIN n := 0;
  REPEAT n := n+1; s[n] := 1; try;
    WHILE NOT good DO
    BEGIN changesequence; try
    END ;
    printsequence
  UNTIL n = maxlength
END .
```

3. Find the smallest positive integer that can be represented as the sum of to cubes (integers raised to the third power) in two different ways.

```
PROGRAM sumofcubes(output);
  VAR i, ih, il, min, a, b, k: integer;
    j, sum, pwr: ARRAY [1..200] OF integer;

  (* pwr[k] = power of k,  sum[k] = p[k] + p[j[k]],
    j[k] = columnindex of last considered candidate in row k,
    ih = rowindex of highest considered row,
    il = rowindex of least still relevant row *)
BEGIN i := 1; il := 1; ih := 2;
  j[1] := 1; pwr[1] := 1; sum[1] := 2;
  j[2] := 1; pwr[2] := 8; sum[2] := 9;
  REPEAT
    min := sum[i]; a := i; b := j[i];
    (* now get next sum in row i *)
    IF j[i] = i THEN
    BEGIN (* there is none left *) il := il+1;
    END ELSE
    BEGIN IF j[i] = 1 THEN
          BEGIN (* the new min was from the first column, now add
            a new row before taking the new sum from the old row *)
            ih := ih + 1; pwr[ih] := ih*ih*ih;
            j[ih] := 1; sum[ih] := pwr[ih]+1;
          END ;
          j[i] := j[i]+1;  (* next candidate in row i *)
          sum[i] := pwr[i] + pwr[j[i]]
    END ;
    (* now find minimal candidate in rows il .. ih *)
    i := il; k := i+1;
    WHILE k <= ih DO
    BEGIN IF sum[k] < sum[i] THEN i := k;  k := k+1
    END
  UNTIL sum[i] = min;
  writeln(min,a,b,i,j[i])
END .
```

4. Find a path of a knight on a chess board which covers all 64 squares.

```
PROGRAM knightstour(output);
CONST n = 5; nsq = 25;
TYPE index = 1..n;
VAR i,j: index;
   q: boolean;
   s: SET OF index;
   a,b: ARRAY [1..8] OF integer;
   h: ARRAY [index, index] OF integer;

PROCEDURE try(i: integer; x,y: index; VAR q: boolean);
   VAR k,u,v: integer; q1: boolean;
BEGIN k := 0;
   REPEAT k := k+1; q1 := false;
      u := x + a[k]; v := y + b[k];
      IF (u IN s) AND (v IN s) THEN
      IF h[u,v] = 0 THEN
      BEGIN h[u,v] := i;
         IF i < nsq THEN
            BEGIN try(i+1,u,v,q1);
               IF NOT q1 THEN h[u,v] := 0
            END ELSE q1 := true
      END
   UNTIL q1 OR (k=8);
   q := q1
END (*try*) ;

BEGIN s := [1,2,3,4,5];
   a[1] :=  2; b[1] :=  1;
   a[2] :=  1; b[2] :=  2;
   a[3] := -1; b[3] :=  2;
   a[4] := -2; b[4] :=  1;
   a[5] := -2; b[5] := -1;
   a[6] := -1; b[6] := -2;
   a[7] :=  1; b[7] := -2;
   a[8] :=  2; b[8] := -1;
   FOR i := 1 TO n DO
      FOR j := 1 TO n DO h[i,j] := 0;
   h[1,1] := 1; try(2,1,1,q);
   IF q THEN
      FOR i := 1 TO n DO
      BEGIN FOR j := 1 TO n DO write(h[i,j]:5);
         writeln
      END
   ELSE writeln(" no solution ")
END .
```

5. Find a solution to the *stable marriage problem.* n men and n women state their preferences of partners. Find n pairs such that no man would prefer to be married to another woman who would also prefer him to her partner. A set of pairs is called *stable*, if no such cases exist [see also Comm. ACM 14, 7, 486-92 (July 71)].

```
PROGRAM marriage(input,output);
CONST n = 8;
TYPE man = 1..n; woman = 1..n; rank = 1..n;
VAR m: man; w: woman; r: rank;
   wmr: ARRAY [man, rank] OF woman;
   mwr: ARRAY [woman, rank] OF man;
   rmw: ARRAY [man, woman] OF rank;
   rwm: ARRAY [woman, man] OF rank;
   x:   ARRAY [man] OF woman;
   y:   ARRAY [woman] OF man;
   single: ARRAY [woman] OF boolean;

PROCEDURE print;
   VAR m: man; rm, rw: integer;
BEGIN rm := 0; rw := 0;
   FOR m := 1 TO n DO
   BEGIN write(x[m]:4);
      rm := rm + rmw[m,x[m]]; rw := rw + rwm[x[m],m]
   END ;
   writeln(rm:8,rw:4);
END (*print*) ;

PROCEDURE try(m: man);
   VAR r: rank; w: woman;

   FUNCTION stable: boolean;
      VAR pm: man; pw: woman;
          i, lim: rank;  s: boolean;
   BEGIN s := true; i := 1;
      WHILE (i<r) AND s DO
      BEGIN pw := wmr[m,i]; i := i+1;
         IF NOT single[pw] THEN s := rwm[pw,m] > rwm[pw,y[pw]]
      END ;
      i := 1; lim := rwm[w,m];
      WHILE (i<lim) AND s DO
      BEGIN pm := mwr[w,i]; i := i+1;
         IF pm < m THEN s := rmw[pm,w] > rmw[pm,x[pm]]
      END ;
      stable := s
   END (*test*) ;

BEGIN (*try*)
   FOR r := 1 TO n DO
   BEGIN w := wmr[m,r];
      IF single[w] THEN
         IF stable THEN
         BEGIN x[m] := w; y[w] := m; single[w] := false;
            IF m < n THEN try(succ(m)) ELSE print;
            single[w] := true
         END
   END
END (*try*) ;
```

```
BEGIN writeln("1");
   FOR m := 1 TO n DO
      FOR r := 1 TO n DO
      BEGIN read(wmr[m,r]); rmw[m,wmr[m,r]] := r
      END ;
   FOR w := 1 TO n DO
      FOR r := 1 TO n DO
      BEGIN read(mwr[w,r]); rwm[w,mwr[w,r]] := r
      END ;
   FOR w := 1 TO n DO single[w] := true;
   try(1)
END .
```

```
5 7 1 2 6 8 4 3
2 3 7 5 4 1 8 6
8 5 1 4 6 2 3 7
3 2 7 4 1 6 8 5
7 2 5 1 3 6 8 4
1 6 7 5 8 4 2 3
2 5 7 6 3 4 8 1
3 8 4 5 7 2 6 1
5 3 7 6 1 2 8 4
8 6 3 5 7 2 1 4
1 5 6 2 4 8 7 3
8 7 3 2 4 1 5 6
6 4 7 3 8 1 2 5
2 8 5 4 6 3 7 1
7 5 2 1 8 6 4 3
7 4 1 5 2 3 6 8
```

6. Find an optimal selection of objects from a given set of n objects under a given constraint. Each object is characterised by two properties v (for value) and w (for weight). The optimal selection is the one with the largest sum of values of its members. The constraint is that the sum of their weights must not surpass a given limit *limv*. The algorithm is called *branch and bound*.

```
PROGRAM selection(input,output);
CONST n = 10;
TYPE index = 1..n;
   object = RECORD v,w: integer END ;
VAR i: index;
   a: ARRAY [index] OF object;
   limw, totv, maxv: integer;
   w1, w2, w3: integer;
   s, opts: SET OF index;
   z: ARRAY [boolean] OF char;

PROCEDURE try(i: index; tw,av: integer);
   VAR av1: integer;
BEGIN (*try inclusion of object i*)
   IF tw + a[i].w <= limw THEN
   BEGIN s := s + [i];
      IF i < n THEN try(i+1, tw+a[i].w, av) ELSE
         IF av > maxv THEN
         BEGIN maxv := av; opts := s
         END ;
      s := s - [i]
   END ;
   (*now try without object i*)  av1 := av - a[i].v;
   IF av1 > maxv THEN
   BEGIN IF i < n THEN try(i+1, tw, av1) ELSE
            BEGIN maxv := av1; opts := s
            END
   END
END (*try*) ;

BEGIN totv := 0;
   FOR i := 1 TO n DO
      WITH a[i] DO
      BEGIN read(w,v); totv := totv + v
      END ;
   read(w1,w2,w3);
   z[true] := "*"; z[false] := " ";
   write(" weight   ");
   FOR i := 1 TO n DO write(a[i].w:4);
   writeln; write(" value    ");
   FOR i := 1 TO n DO write(a[i].v:4);
   writeln;
   REPEAT limw := w1; maxv := 0; s := []; opts := [];
      try(1,0,totv);
      write(limw);
      FOR i := 1 TO n DO write("   ", z[i IN opts]);
      writeln; w1 := w1 + w2
   UNTIL w1 > w3
END .
```

10. List and tree structures, pointers.

1. A procedure *search* is to locate records with a given key in an ordered list. If the key is not present, then a new record is to be inserted so that the ordering of keys is maintained. Use a sentinel at the end of the list.

```
PROGRAM list(input,output);
  TYPE ref = ↑word;
    word = RECORD key: integer;
                  count: integer;
                  next: ref
           END ;
  VAR k: integer; root, sentinel: ref;

  PROCEDURE search(x: integer; VAR root: ref);
    VAR w1,w2,w3: ref;
  BEGIN w2 := root; w1 := w2↑.next; sentinel↑.key := x;
    WHILE w1↑.key < x DO
      BEGIN w2 := w1; w1 := w2↑.next
      END ;
    IF (w1↑.key = x) AND (w1 # sentinel) THEN
      w1↑.count := w1↑.count + 1 ELSE
    BEGIN new(w3); (*insert w3 between w1 AND w2*)
      WITH w3↑ DO
        BEGIN key := x; count := 1; next := w1
        END ;
      w2↑.next := w3
    END
  END (*search*) ;

  PROCEDURE printlist(w,z: ref);
  BEGIN WHILE w # z DO
      BEGIN writeln(w↑.key, w↑.count);
        w := w↑.next
      END
  END (*printlist*) ;

BEGIN new(root); new(sentinel); root↑.next := sentinel;
  read(k);
  WHILE k # 0 DO
    BEGIN search(k, root); read(k)
    END ;
  printlist(root↑.next,sentinel)
END .
```

2. Instead of keeping the list ordered according to keys, reorder it as follows: After each search, the accessed record is moved to the top of the list. In this case, repeated accesses to the same element will be very fast. Use a sentinel at the end of the list.

```
PROGRAM list(input,output);
  TYPE ref = ↑word;
    word = RECORD key: integer;
                  count: integer;
                  next: ref
           END ;
  VAR k: integer; root, sentinel: ref;

  PROCEDURE search(x: integer; VAR root: ref);
    VAR w1,w2: ref;
  BEGIN w1 := root; sentinel↑.key := x;
    IF w1 = sentinel THEN
    BEGIN (*first element*) new(root);
      WITH root↑ DO
        BEGIN key := x; count := 1; next := sentinel
        END
    END ELSE
    IF w1↑.key = x THEN w1↑.count := w1↑.count + 1 ELSE
    BEGIN (*search*)
      REPEAT w2 := w1; w1 := w2↑.next
      UNTIL w1↑.key = x;
      IF w1 = sentinel THEN
      BEGIN (*insert*)
        w2 := root; new(root);
        WITH root↑ DO
          BEGIN key := x; count := 1; next := w2
          END
      END ELSE
      BEGIN (*found, now reorder*)
        w1↑.count := w1↑.count + 1;
        w2↑.next := w1↑.next; w1↑.next := root; root := w1
      END
    END
  END (*search*) ;

  PROCEDURE printlist(w,z: ref);
  BEGIN WHILE w # z DO
      BEGIN writeln(w↑.key, w↑.count);
        w := w↑.next
      END
  END (*printlist*) ;

BEGIN new(sentinel); root := sentinel;
  read(k);
  WHILE k # 0 DO
    BEGIN search(k, root); read(k)
    END ;
  printlist(root,sentinel)
END .
```

3. Read a sequence of relations defining a directed, finite graph. Then establish whether or not a partial ordering is defined. If so, print the elements in a sequence showing the partial ordering. (Topological sorting).

```
PROGRAM topsort(input,output);
TYPE lref = ↑leader;
   tref = ↑trailer;
   leader = RECORD key: integer;
               count: integer;
               trail: tref;
               next:  lref;
            END ;
   trailer= RECORD id: lref;
               next: tref
            END ;
VAR head, tail, p,q: lref;
   t: tref; z: integer;
   x,y: integer;

FUNCTION l(w: integer): lref;
   (*reference to leader with key w*)
   VAR h: lref;
BEGIN h := head; tail↑.key := w;  (*sentinel*)
   WHILE h↑.key # w DO  h := h↑.next;
   IF h = tail THEN
      BEGIN (*no element with key w in the list*)
         new(tail); z := z+1;
         h↑.count := 0; h↑.trail := NIL; h↑.next := tail
      END ;
   l := h
END (*l*) ;

BEGIN (*initialise list of leaders with a dummy*)
   new(head); tail := head; z := 0;

(*input phase*)  read(x);
   WHILE x # 0 DO
   BEGIN read(y); writeln(x,y);
      p := l(x); q := l(y);
      new(t); t↑.id := q; t↑.next := p↑.trail;
      p↑.trail := t; q↑.count := q↑.count + 1;
      read(x)
   END ;

(*search for leaders with count = 0*)
   p := head; head := NIL;
   WHILE p # tail DO
   BEGIN q := p; p := p↑.next;
      IF q↑.count = 0 THEN
         BEGIN q↑.next := head; head := q
         END ;
   END ;

(*output phase*)  q := head;
   WHILE q # NIL DO
   BEGIN writeln(q↑.key); z := z-1;
      t := q↑.trail; q := q↑.next;
      WHILE t # NIL DO
```

```
      BEGIN p := t↑.id; p↑.count := p↑.count - 1;
        IF p↑.count = 0 THEN
        BEGIN (*insert p↑ in q-list*)
          p↑.next := q; q := p
        END ;
        t := t↑.next
      END
    END ;

  IF z # 0 THEN writeln(" this set is not partially ordered")
END .
```

4. Insertion and deletion in a binary tree. Read a sequence of integers. A positive integer signifies that it should be inserted in an ordered binary tree as the key of a node. A negative integer signifies that a node with its absolute value as key should be searched and deleted.

```
PROGRAM tree(input,output);
TYPE ref = ↑word;
   word = RECORD key: integer;
            count: integer;
            left, right: ref;
          END ;
VAR root: ref; k: integer;

PROCEDURE printtree(w: ref; l: integer);
   VAR i: integer;
BEGIN IF w # NIL THEN
      WITH w↑ DO
      BEGIN printtree(left, l+1);
            FOR i := 1 TO l DO write("    ");
            writeln(key);
            printtree(right, l+1)
      END
END ;

PROCEDURE search(x: integer; VAR p: ref);
BEGIN
   IF p = NIL THEN
   BEGIN (*word is not in tree; insert it*)
      new(p);
      WITH p↑ DO
         BEGIN key := x; count := 1; left := NIL; right := NIL
         END
   END ELSE
   IF x < p↑.key THEN search(x, p↑.left) ELSE
   IF x > p↑.key THEN search(x, p↑.right) ELSE
      p↑.count := p↑.count + 1
END (*search*) ;

PROCEDURE delete(x: integer; VAR p: ref);
   VAR q: ref;

   PROCEDURE del(VAR r: ref);
   BEGIN IF r↑.right # NIL THEN del(r↑.right) ELSE
         BEGIN q↑.key := r↑.key; q↑.count := r↑.count;
            q := r; r := r↑.left
         END
   END ;

BEGIN (*delete*)
   IF p = NIL THEN writeln(" word is not in tree") ELSE
   IF x < p↑.key THEN delete(x, p↑.left) ELSE
   IF x > p↑.key THEN delete(x, p↑.right) ELSE
   BEGIN (*delete p↑*)  q := p;
      IF q↑.right = NIL THEN p := q↑.left ELSE
      IF q↑.left  = NIL THEN p := q↑.right ELSE del(q↑.left);
      (*dispose(q)*)
   END
END (*delete*) ;
```

```
BEGIN root := NIL; read(k);
   WHILE k # 0 DO
   BEGIN IF k > 0 THEN
      BEGIN writeln(" insert", k); search(k,root)
      END ELSE
      BEGIN writeln(" delete",-k); delete(-k,root)
      END ;
      printtree(root,0); read(k)
   END ;
END .
```

5. Insertion and deletion in a *AVL-balanced* tree. In the previous program, the binary tree may grow in all sorts of shapes -- if the inserted keys are ordered upon arrival, the "tree" even degenerates into a linear list. In the following program, a balance is maintained, such that at each node the heights of its two subtrees differ by at most 1.

```
PROGRAM baltree(input,output);
TYPE ref = ↑word;
   word = RECORD key: integer;
             count: integer;
             left, right: ref;
             bal: -1..+1
          END ;
VAR root: ref;  h: boolean; k: integer;

PROCEDURE printtree(w: ref; l: integer);
   VAR i: integer;
BEGIN IF w # NIL THEN
      WITH w↑ DO
      BEGIN printtree(left, l+1);
            FOR i := 1 TO l DO write("    ");
            writeln(key:5, bal:3);
            printtree(right, l+1)
      END
END ;

PROCEDURE search(x: integer; VAR p: ref; VAR h: boolean);
   VAR p1,p2: ref;   (*h = false*)
BEGIN
   IF p = NIL THEN
   BEGIN (*word is not in tree; insert it*)
      new(p); h := true;
      WITH p↑ DO
      BEGIN key := x; count := 1;
         left := NIL; right := NIL; bal := 0
      END
   END ELSE
   IF x < p↑.key THEN
   BEGIN search(x, p↑.left, h);
      IF h THEN    (*left branch has grown higher*)
      CASE p↑.bal OF
      1: BEGIN p↑.bal := 0; h := false
         END ;
      0: p↑.bal := -1;
     -1: BEGIN (*rebalance*) p1 := p↑.left;
            IF p1↑.bal = -1 THEN
            BEGIN (*single LL rotation*)
               p↑.left := p1↑.right; p1↑.right := p;
               p↑.bal := 0; p := p1
            END ELSE
            BEGIN (*double LR rotation*)  p2 := p1↑.right;
               p1↑.right := p2↑.left; p2↑.left := p1;
               p↑.left := p2↑.right; p2↑.right := p;
               IF p2↑.bal = -1 THEN p↑.bal := +1 ELSE p↑.bal := 0;
               IF p2↑.bal = +1 THEN p1↑.bal := -1 ELSE p1↑.bal := 0;
               p := p2
            END ;
            p↑.bal := 0; h := false
         END
```

```
      END
    END ELSE
    IF x > p↑.key THEN
    BEGIN search(x, p↑.right, h);
      IF h THEN    (*right branch has grown higher*)
      CASE p↑.bal OF
     -1: BEGIN p↑.bal := 0; h := false
         END ;
      0: p↑.bal := +1;
      1: BEGIN (*rebalance*) p1 := p↑.right;
           IF p1↑.bal = +1 THEN
           BEGIN (*single RR rotation*)
             p↑.right := p1↑.left; p1↑.left := p;
             p↑.bal := 0; p := p1
           END ELSE
           BEGIN (*double RL rotation*)  p2 := p1↑.left;
             p1↑.left := p2↑.right; p2↑.right := p1;
             p↑.right := p2↑.left; p2↑.left := p;
             IF p2↑.bal = +1 THEN p↑.bal := -1 ELSE p↑.bal := 0;
             IF p2↑.bal = -1 THEN p1↑.bal := +1 ELSE p1↑.bal := 0;
             p := p2
           END ;
           p↑.bal := 0; h := false
         END
      END
    END
    ELSE
    BEGIN p↑.count := p↑.count + 1; h := false
    END
  END (*search*) ;

  PROCEDURE delete(x: integer; VAR p: ref; VAR h: boolean);
    VAR q: ref;    (*h = false*)

    PROCEDURE balance1(VAR p: ref; VAR h: boolean);
      VAR p1,p2: ref; b1,b2: -1..+1;
    BEGIN (*h = true, left branch has become less high*)
      CASE p↑.bal OF
     -1: p↑.bal := 0;
      0: BEGIN p↑.bal := +1; h := false
         END ;
      1: BEGIN (*rebalance*) p1 := p↑.right; b1 := p1↑.bal;
           IF b1 >= 0 THEN
           BEGIN (*single RR rotation*)
             p↑.right := p1↑.left; p1↑.left := p;
             IF b1 = 0 THEN
             BEGIN p↑.bal := +1; p1↑.bal := -1; h := false
             END ELSE
             BEGIN p↑.bal := 0; p1↑.bal := 0
             END ;
             p := p1
           END ELSE
           BEGIN (*double RL rotation*)
             p2 := p1↑.left; b2 := p2↑.bal;
             p1↑.left := p2↑.right; p2↑.right := p1;
             p↑.right := p2↑.left; p2↑.left := p;
             IF b2 = +1 THEN p↑.bal := -1 ELSE p↑.bal := 0;
             IF b2 = -1 THEN p1↑.bal := +1 ELSE p1↑.bal := 0;
```

```
            p := p2; p2↑.bal := 0
          END
        END
      END
    END (*balance1*) ;

    PROCEDURE balance2(VAR p: ref; VAR h: boolean);
      VAR p1,p2: ref; b1,b2: -1..+1;
    BEGIN (*h = true, right branch has become less high*)
      CASE p↑.bal OF
       1: p↑.bal := 0;
       0: BEGIN p↑.bal := -1; h := false
          END ;
      -1: BEGIN (*rebalance*) p1 := p↑.left; b1 := p1↑.bal;
            IF b1 <= 0 THEN
            BEGIN (*single LL rotation*)
              p↑.left := p1↑.right; p1↑.right := p;
              IF b1 = 0 THEN
              BEGIN p↑.bal := -1; p1↑.bal := +1; h := false
              END ELSE
              BEGIN p↑.bal := 0; p1↑.bal := 0
              END ;
              p := p1
            END ELSE
            BEGIN (*double LR rotation*)
              p2 := p1↑.right; b2 := p2↑.bal;
              p1↑.right := p2↑.left; p2↑.left := p1;
              p↑.left := p2↑.right; p2↑.right := p;
              IF b2 = -1 THEN p↑.bal := +1 ELSE p↑.bal := 0;
              IF b2 = +1 THEN p1↑.bal := -1 ELSE p1↑.bal := 0;
              p := p2; p2↑.bal := 0
            END
          END
        END
    END (*balance2*) ;

    PROCEDURE del(VAR r: ref; VAR h: boolean);
    BEGIN (*h = false*)
      IF r↑.right # NIL THEN
      BEGIN del(r↑.right,h); IF h THEN balance2(r,h)
      END ELSE
      BEGIN q↑.key := r↑.key; q↑.count := r↑.count;
          r := r↑.left; h := true
      END
    END ;

BEGIN (*delete*)
    IF p = NIL THEN
      BEGIN writeln(" key is not in tree"); h := false
      END ELSE
    IF x < p↑.key THEN
      BEGIN delete(x,p↑.left,h); IF h THEN balance1(p,h)
      END ELSE
    IF x > p↑.key THEN
      BEGIN delete(x,p↑.right,h); IF h THEN balance2(p,h)
      END ELSE
    BEGIN (*delete p↑*)  q := p;
      IF q↑.right = NIL THEN
```

```
            BEGIN p := q↑.left; h := true
            END ELSE
         IF q↑.left = NIL THEN
            BEGIN p := q↑.right; h := true
            END ELSE
         BEGIN del(q↑.left,h);
            IF h THEN balance1(p,h)
         END ;
         (*dispose(q)*)
      END
   END (*delete*) ;

   BEGIN read(k); root := NIL;
      WHILE k # 0 DO
      BEGIN IF k >= 0 THEN
         BEGIN writeln(" insert", k); search( k,root,h)
         END ELSE
         BEGIN writeln(" delete",-k); delete(-k,root,h)
         END ;
         printtree(root,0); read(k)
      END ;
   END .
```

6. Insert and delete elements in a B-tree of page size 2n. Read a sequence of keys; positive values denote insertion, negative ones deletion. Print the resulting B-tree after each operation.

```
PROGRAM Btree(input,output);
CONST n = 2; nn = 4;  (*page size*)
TYPE ref = ↑page;
   item = RECORD key: integer;
             p: ref;
             count: integer;
          END ;
   page = RECORD m: 0..nn;  (*no. of items*)
             p0: ref;
             e : ARRAY [1..nn] OF item;
          END ;
VAR root, q: ref; x: integer;
   h: boolean;  u: item;

PROCEDURE printtree(p: ref; l: integer);
   VAR i: integer;
BEGIN IF p # NIL THEN
   WITH p↑ DO
   BEGIN FOR i := 1 TO l DO write("    ");
         FOR i := 1 TO m DO write(e[i].key: 4);
         writeln;
         printtree(p0,l+1);
         FOR i := 1 TO m DO printtree(e[i].p, l+1)
   END
END ;

PROCEDURE search(x: integer; a:ref;
                 VAR h: boolean; VAR v: item);
(*search key x on B-tree with root a; if found, increment counter.  Otherwise
insert an item with key x and count 1 in tree.  If an item emerges to be passed
to a lower level, then assign it to v;  h := "tree a has become higher"*)

   VAR k,l,r: integer; q: ref; u: item;

   PROCEDURE insert;
      VAR i: integer; b: ref;
   BEGIN (*insert u to the right of a↑.e[r]*)
      WITH a↑ DO
      BEGIN IF m < nn THEN
         BEGIN m := m+1; h := false;
            FOR i := m DOWNTO r+2 DO e[i] := e[i-1];
            e[r+1] := u
         END ELSE
         BEGIN (*page a↑ is full; split it and assign the emerging
               item to v*)  new(b);
            IF r <= n THEN
            BEGIN IF r = n THEN v := u ELSE
                  BEGIN v := e[n];
                     FOR i := n DOWNTO r+2 DO e[i] := e[i-1];
                     e[r+1] := u
                  END ;
               FOR i := 1 TO n DO b↑.e[i] := a↑.e[i+n];
            END ELSE
```

```
            BEGIN (*insert u in right page*)  r := r-n; v := e[n+1];
               FOR i := 1 TO r-1 DO bt.e[i] := at.e[i+n+1];
               bt.e[r] := u;
               FOR i := r+1 TO n DO bt.e[i] := at.e[i+n]
            END ;
            m := n; bt.m := n; bt.p0 := v.p; v.p := b;
         END
      END (*WITH*)
   END (*insert*) ;

BEGIN (*search key x on page at; h = false*)
   IF a = NIL THEN
   BEGIN (*item with key x is not in tree*) h := true;
      WITH v DO
      BEGIN key := x; count := 1; p := NIL
      END
   END ELSE
   WITH at DO
   BEGIN l := 1; r := m;  (*binary array search*)
      REPEAT k := (l+r) DIV 2;
         IF x <= e[k].key THEN r := k-1;
         IF x >= e[k].key THEN l := k+1;
      UNTIL r < l;
      IF l-r > 1 THEN
      BEGIN (*found*) e[k].count := e[k].count + 1; h := false
      END ELSE
      BEGIN (*item is not on this page*)
         IF r = 0 THEN q := p0 ELSE q := e[r].p;
         search(x,q,h,u); IF h THEN insert
      END
   END
END (*search*) ;

PROCEDURE delete(x: integer; a: ref; VAR h: boolean);
(*search and delete key x in B-tree a; if a page underflow is  necessary,
balance with adjacent page if possible, otherwise merge;  h := "page a is
undersize"*)

   VAR i,k,l,r: integer; q: ref;

   PROCEDURE underflow(c,a: ref; s: integer; VAR h: boolean);
      (*a = underflow page, c = ancestor page*)
      VAR b: ref; i,k,mb,mc: integer;
   BEGIN mc := ct.m;  (*h = true, at.m = n-1*)
      IF s < mc THEN
      BEGIN (*b := page to the right of a*)  s := s+1;
         b := ct.e[s].p; mb := bt.m; k := (mb-n+1) DIV 2;
         (*k = no. of items available on adjacent page b*)
         at.e[n] := ct.e[s]; at.e[n].p := bt.p0;
         IF k > 0 THEN
         BEGIN (*move k items from b to a*)
            FOR i := 1 TO k-1 DO at.e[i+n] := bt.e[i];
            ct.e[s] := bt.e[k]; ct.e[s].p := b;
            bt.p0 := bt.e[k].p; mb := mb-k;
            FOR i := 1 TO mb DO bt.e[i] := bt.e[i+k];
            bt.m := mb; at.m := n-l+k; h := false
         END ELSE
         BEGIN (*merge pages a and b*)
```

```
        FOR i := 1 TO n DO a↑.e[i+n] := b↑.e[i];
        FOR i := s TO mc-1 DO c↑.e[i] := c↑.e[i+1];
        a↑.m := nn; c↑.m := mc-1; (*dispose(b)*)
      END
    END ELSE
    BEGIN (*b := page to the left of a*)
      IF s = 1 THEN b := c↑.p0 ELSE b := c↑.e[s-1].p;
      mb := b↑.m + 1; k := (mb-n) DIV 2;
      IF k > 0 THEN
      BEGIN (*move k items from page b to a*)
        FOR i := n-1 DOWNTO 1 DO a↑.e[i+k] := a↑.e[i];
        a↑.e[k] := c↑.e[s]; a↑.e[k].p := a↑.p0; mb := mb-k;
        FOR i := k-1 DOWNTO 1 DO a↑.e[i] := b↑.e[i+mb];
        a↑.p0 := b↑.e[mb].p;
        c↑.e[s] := b↑.e[mb]; c↑.e[s].p := a;
        b↑.m := mb-1; a↑.m := n-1+k; h := false
      END ELSE
      BEGIN (*merge pages a and b*)
        b↑.e[mb] := c↑.e[s]; b↑.e[mb].p := a↑.p0;
        FOR i := 1 TO n-1 DO b↑.e[i+mb] := a↑.e[i];
        b↑.m := nn; c↑.m := mc-1; (*dispose(a)*)
      END
    END
  END (*underflow*) ;

  PROCEDURE del(p: ref; VAR h: boolean);
    VAR q: ref;   (*global a,k*)
  BEGIN
    WITH p↑ DO
    BEGIN q := e[m].p;
      IF q # NIL THEN
      BEGIN del(q,h); IF h THEN underflow(p,q,m,h)
      END ELSE
      BEGIN p↑.e[m].p := a↑.e[k].p; a↑.e[k] := p↑.e[m];
        m := m-1; h := m<n
      END
    END
  END (*del*) ;

BEGIN (*delete*)
  IF a = NIL THEN
  BEGIN writeln(" key is not in tree"); h := false
  END ELSE
  WITH a↑ DO
  BEGIN l := 1; r := m;  (*binary array search*)
    REPEAT k := (l+r) DIV 2;
      IF x <= e[k].key THEN r := k-1;
      IF x >= e[k].key THEN l := k+1;
    UNTIL l > r;
    IF r=0 THEN q := p0 ELSE q := e[r].p;
    IF l-r > 1 THEN
    BEGIN (*found, now delete e[k]*)
      IF q = NIL THEN
      BEGIN (*a is a terminal page*)  m := m-1; h := m<n;
        FOR i := k TO m DO e[i] := e[i+1];
      END ELSE
      BEGIN del(q,h); IF h THEN underflow(a,q,r,h)
      END
```

```
      END ELSE
      BEGIN delete(x,q,h); IF h THEN underflow(a,q,r,h)
      END
   END
END (*delete*) ;


BEGIN root := NIL; read(x);
   WHILE x # 0 DO
   BEGIN writeln(" search key", x);
      search(x,root,h,u);
      IF h THEN
      BEGIN (*insert new base page*) q := root; new(root);
         WITH root↑ DO
            BEGIN m := 1; p0 := q; e[1] := u
            END
      END ;
      printtree(root,1); read(x)
   END ;
   read(x);
   WHILE x # 0 DO
   BEGIN writeln(" delete key", x);
      delete(x,root,h);
      IF h THEN
      BEGIN (*base page size was reduced*)
         IF root↑.m = 0 THEN
         BEGIN q := root; root := q↑.p0; (*dispose(q)*)
         END ;
      END ;
      printtree(root,1); read(x)
   END
END .
```

7. Find the optimally structured binary search tree for n keys. Known are the search frequencies of the keys, b[i] for key[i], and the frequencies of searches with arguments that are not keys (represented in the tree). a[i] is the frequency of an argument lying between key[i-1] and key[i]. Use Knuth's algorithm, *Acta Informatica* 1, 1, 14-25 (1971). The following example uses Pascal keywords as keys.

```
PROGRAM optimaltree(input,output);
CONST n = 31; (*no. of keys*)
   kln = 10;  (*max keylength*)
TYPE index = 0..n;
   alfa = PACKED ARRAY [1..kln] OF char;
VAR ch: char;
      k1, k2: integer;
      id: alfa;                          (*identifier or key*)
      buf: ARRAY [1..kln] OF char;       (*character buffer*)
      key: ARRAY [1..n] OF alfa;
      i,j,k: integer;
      a: ARRAY [1..n] OF integer;
      b: ARRAY [index] OF integer;
      p,w: ARRAY [index,index] OF integer;
      r: ARRAY [index,index] OF index;
      suma, sumb: integer;

FUNCTION baltree(i,j: index): integer;
   VAR k: integer;
BEGIN k := (i+j+1) DIV 2; r[i,j] := k;
   IF i >= j THEN baltree := b[k] ELSE
      baltree := baltree(i,k-1) + baltree(k,j) + w[i,j]
END (*baltree*) ;

PROCEDURE opttree;
   VAR x, min: integer;
      i,j,k,h,m: index;
BEGIN (*argument: w, result: p,r*)
   FOR i := 0 TO n DO p[i,i] := w[i,i];    (*width of tree h = 0*)
   FOR i := 0 TO n-1 DO                    (*width of tree h = 1*)
   BEGIN j := i+1;
      p[i,j] := p[i,i] + p[j,j];  r[i,j] := j
   END ;
   FOR h := 2 TO n DO                      (* h = width of considered tree *)
   FOR i := 0 TO n-h DO                    (* i = left index of considered tree *)
   BEGIN  j := i+h;                        (* j = right index of considered tree *)
      m := r[i,j-1];  min := p[i,m-1] + p[m,j];
      FOR k := m+1 TO r[i+1,j] DO
      BEGIN x := p[i,k-1] + p[k,j];
         IF x < min THEN
            BEGIN m := k;  min := x
            END
      END ;
      p[i,j] := min + w[i,j];  r[i,j] := m
   END ;
END (*opttree*) ;

PROCEDURE printtree;
   CONST lw = 120;                         (*line width of printer*)
   TYPE ref = ↑node;
      lineposition = 0..lw;
      node = RECORD key: alfa;
```

```
                pos: lineposition;
                left, right, link: ref
            END ;
    VAR root, current, next: ref;
        q,q1,q2: ref;
        i: integer;
        k: integer;
        u, u1, u2, u3, u4: lineposition;

    FUNCTION tree(i,j: index): ref;
        VAR p: ref;
    BEGIN IF i = j THEN p := NIL ELSE
        BEGIN new(p);
            p↑.left := tree(i, r[i,j]-1);
            p↑.pos := trunc((lw-kln)*k/(n-1)) + (kln DIV 2);  k := k+1;
            p↑.key := key[r[i,j]];
            p↑.right := tree(r[i,j], j)
        END ;
        tree := p
    END ;

BEGIN k := 0;  root := tree(0,n);
    current := root; root↑.link := NIL;
    next := NIL;
    WHILE current # NIL DO
    BEGIN (*proceed down; first write vertical lines*)
        FOR i := 1 TO 3 DO
        BEGIN u := 0; q := current;
            REPEAT u1 := q↑.pos;
                REPEAT write(" "); u := u+1
                UNTIL u = u1;
                write(":"); u := u+1; q := q↑.link
            UNTIL q = NIL;
            writeln
        END ;
        (*now print master line; descending from nodes on current list
         collect their descendants and form next list*)
        q := current; u := 0;
        REPEAT unpack(q↑.key, buf, 1);
            (*center key about pos*)  i := kln;
            WHILE buf[i] = " " DO  i := i-1;
            u2 := q↑.pos - ((i-1) DIV 2); u3 := u2+i;
            q1 := q↑.left; q2 := q↑.right;
            IF q1 = NIL THEN u1 := u2 ELSE
                BEGIN u1 := q1↑.pos; q1↑.link := next; next := q1
                END ;
            IF q2 = NIL THEN u4 := u3 ELSE
                BEGIN u4 := q2↑.pos+1; q2↑.link := next; next := q2
                END ;
            i := 0;
            WHILE u < u1 DO BEGIN write(" "); u := u+1 END ;
            WHILE u < u2 DO BEGIN write("-"); u := u+1 END ;
            WHILE u < u3 DO BEGIN i := i+1; write(buf[i]); u := u+1 END ;
            WHILE u < u4 DO BEGIN write("-"); u := u+1 END ;
            q := q↑.link
        UNTIL q = NIL;
        writeln;
        (*now invert next list AND make it current list*)
```

```
      current := NIL;
      WHILE next # NIL DO
        BEGIN q := next; next := q↑.link;
          q↑.link := current; current := q
        END
    END
  END (*printtree*) ;

BEGIN (*initialize table of keys and counters*)
  key[ 1] := "ARRAY   ";  key[ 2] := "BEGIN   ";      key[ 3] := "CASE    ";
  key[ 4] := "CONST   ";  key[ 5] := "DIV     ";      key[ 6] := "DOWNTO  ";
  key[ 7] := "DO      ";  key[ 8] := "ELSE    ";      key[ 9] := "END     ";
  key[10] := "FILE    ";  key[11] := "FOR     ";      key[12] := "FUNCTION ";
  key[13] := "GOTO    ";  key[14] := "IF      ";      key[15] := "IN      ";
  key[16] := "LABEL   ";  key[17] := "MOD     ";      key[18] := "NIL     ";
  key[19] := "OF      ";  key[20] := "PROCEDURE ";    key[21] := "PROGRAM ";
  key[22] := "RECORD  ";  key[23] := "REPEAT  ";      key[24] := "SET     ";
  key[25] := "THEN    ";  key[26] := "TO      ";      key[27] := "TYPE    ";
  key[28] := "UNTIL   ";  key[29] := "VAR     ";      key[30] := "WHILE   ";
  key[31] := "WITH    ";
  FOR i := 1 TO n DO
    BEGIN a[i] := 0;  b[i] := 0
    END ;
  b[0] := 0; k2 := kln;
  (*scan input text and determine a and b*)
  WHILE NOT eof(input) DO
  BEGIN read(ch);
    IF ch IN ["a".."z"] THEN
    BEGIN (*identifier or key*) k1 := 0;
      REPEAT IF k1 < kln THEN
          BEGIN k1 := k1+1; buf[k1] := ch
          END ;
        read(ch)
      UNTIL NOT (ch IN ["a".."z", "0".."9"]);
      IF k1 >= k2 THEN k2 := k1 ELSE
      REPEAT buf[k2] := " "; k2 := k2-1
      UNTIL k2 = k1;
      pack(buf,1,id);
      i := 1; j := n;
      REPEAT k := (i+j) DIV 2;
        IF key[k] <= id THEN i := k+1;
        IF key[k] >= id THEN j := k-1;
      UNTIL i > j;
      IF key[k] = id THEN a[k] := a[k] + 1 ELSE
        BEGIN k := (i+j) DIV 2; b[k] := b[k]+1
        END ;
    END ELSE
    IF ch = """" THEN
      REPEAT read(ch) UNTIL ch = """" ELSE
    IF ch = "(*" THEN
      REPEAT read(ch) UNTIL ch = "*)"
  END ;
  writeln(" keys and frequencies of occurrence:");
  suma := 0; sumb := b[0];
  FOR i := 1 TO n DO
  BEGIN suma := suma+a[i]; sumb := sumb+b[i];
    writeln(b[i-1], a[i], " ", key[i])
  END ;
```

```
    writeln(b[n]);
    writeln("      ------      ------");
    writeln(sumb, suma);

    (*compute w from a and b*)
    FOR i := 0 TO n DO
    BEGIN w[i,i] := b[i];
       FOR j := i+1 TO n DO w[i,j] := w[i,j-1] + a[j] + b[j]
    END ;
    writeln;
    write(" average path length of balanced tree =");
    writeln(baltree(0,n)/w[0,n]:6:3); printtree;

    opttree;
    writeln;
    write(" average path length of optimal tree =");
    writeln(p[0,n]/w[0,n]:6:3);  printtree;

    (*now consider keys only, setting b = 0*)
    FOR i := 0 TO n DO
    BEGIN w[i,i] := 0;
       FOR j := i+1 TO n DO w[i,j] := w[i,j-1] + a[j]
    END ;
    opttree;
    writeln;
    writeln(" optimal tree considering keys only");
    printtree
END .
```

11. Cross reference generators

1. Read a text and generate a cross reference table of all words, i.e. sequences of characters that begin with a letter and consist of letters and digits only. Blanks, ends of lines, and special characters are considered to be separators. Use a binary tree to store the words encountered.

```
PROGRAM crossref(f,output);
CONST c1 = 10;   (*length of words*)
      c2 =  8;   (*numbers per line*)
      c3 =  6;   (*digits per number*)
      c4 = 9999; (*max line number*)
TYPE alfa = PACKED ARRAY [1..c1] OF char;
   wordref = ↑word;
      itemref = ↑item;
      word = RECORD key: alfa;
                    first, last: itemref;
                    left, right: wordref
             END ;
      item = PACKED RECORD
                    lno: 0..9999;
                    next: itemref
             END ;
VAR root: wordref;
      k,k1: integer;
      n: integer;          (*current line number*)
      id: alfa;
      f: text;
      a: ARRAY [1..c1] OF char;

PROCEDURE search(VAR w1: wordref);
   VAR w: wordref; x: itemref;
BEGIN w := w1;
   IF w = NIL THEN
   BEGIN new(w); new(x);
      WITH w↑ DO
      BEGIN key := id; left := NIL; right := NIL;
         first := x; last := x
      END ;
      x↑.lno := n; x↑.next := NIL; w1 := w
   END ELSE
   IF id < w↑.key THEN search(w↑.left) ELSE
   IF id > w↑.key THEN search(w↑.right) ELSE
   BEGIN new(x); x↑.lno := n; x↑.next := NIL;
      w↑.last↑.next := x; w↑.last := x
   END
END (*search*) ;

PROCEDURE printtree(w: wordref);

   PROCEDURE printword(w: word);
      VAR l: integer; x: itemref;
   BEGIN write(" ", w.key);
      x := w.first; l := 0;
      REPEAT IF l = c2 THEN
             BEGIN writeln;
                l := 0; write(" ":c1+1)
             END ;
```

```
        I := I+1; write(x↑.lno:c3); x := x↑.next
      UNTIL x = NIL;
      writeln
    END (*printword*) ;

  BEGIN IF w # NIL THEN
    BEGIN printtree(w↑.left);
        printword(w↑); printtree(w↑.right)
    END
  END (*printtree*) ;

  BEGIN root := NIL; n := 0; k1 := c1;
    page(output); reset(f);
    WHILE NOT eof(f) DO
    BEGIN IF n = c4 THEN n := 0;
      n := n+1; write(n:c3);       (*next line*)
      write(" ");
      WHILE NOT eoln(f) DO
      BEGIN (*scan non-empty line*)
        IF f↑ IN ["a".."z"] THEN
        BEGIN k := 0;
          REPEAT IF k < c1 THEN
            BEGIN k := k+1;  a[k] := f↑;
            END ;
            write(f↑); get(f)
          UNTIL NOT (f↑ IN ["a".."z","0".."9"]);
          IF k >= k1 THEN k1 := k ELSE
          REPEAT a[k1] := " ";  k1 := k1-1
          UNTIL k1 = k;
          pack(a,1,id); search(root)
        END ELSE
        BEGIN (*check for quote or comment*)
          IF f↑ = """" THEN
            REPEAT write(f↑); get(f)
            UNTIL f↑ = """" ELSE
          IF f↑ = "{" THEN
            REPEAT write(f↑); get(f)
            UNTIL f↑ = "}" ;
          write(f↑); get(f)
        END
      END ;
      writeln; get(f)
    END ;

    page(output); printtree(root);
  END .
```

2. Cross reference generator as above, but using a hash table instead of a binary tree to store the words encountered.

```
PROGRAM crossref(f,output);
LABEL 13;
CONST c1 = 10;   (*length of words*)
      c2 = 8;    (*numbers per line*)
      c3 =  6;   (*digits per number*)
      c4 = 9999; (*max line number*)
      p = 997;   (*prime number*)
      free = "          ";
TYPE  index = 0..p;
      itemref = ↑item;
      word = RECORD key: alfa;
                    first, last: itemref;
                    fol: index
             END ;
      item = PACKED RECORD
                    lno: 0..9999;
                    next: itemref
             END ;
VAR i, top: index;
      k,k1: integer;
      n: integer;          (*current line number*)
      id: alfa;
      f: text;
      a: ARRAY [1..c1] OF char;
      letters, letdigs: SET OF char;
      t: ARRAY [0..p] OF word;       (*hash table*)

PROCEDURE search;
   VAR h,d,i: index;
      x: itemref; f: boolean;
   (*global variables: t, id, top*)
BEGIN h := ord(id) DIV 4096 MOD p;
   (*Pascal-6000 defines ord on packed character array of length 10.
     Division is needed because division operates on 48 bits only! *)
   f := false; d := 1;
   new(x); x↑.lno := n; x↑.next := NIL;
   REPEAT
      IF t[h].key = id THEN
         BEGIN (*found*) f := true;
            t[h].last↑.next := x; t[h].last := x
         END ELSE
      IF t[h].key = free THEN
         BEGIN (*new entry*) f := true;
            WITH t[h] DO
            BEGIN key := id; first := x; last := x; fol := top
            END ;
            top := h
         END ELSE
         BEGIN (*collision*) h := h+d; d := d+2;
            IF h >= p THEN h := h-p;
            IF d = p THEN
               BEGIN writeln(" table overflow"); GOTO 13
               END
         END
   UNTIL f
```

```
END (*search*) ;

PROCEDURE printtable;
   VAR i,j,m: index;

   PROCEDURE printword(w: word);
      VAR l: integer; x: itemref;
   BEGIN write(" ", w.key);
      x := w.first; l := 0;
      REPEAT IF l = c2 THEN
            BEGIN writeln;
               l := 0; write(" ":c1+1)
            END ;
         l := l+1; write(x↑.lno:c3); x := x↑.next
      UNTIL x = NIL;
      writeln
   END (*printword*) ;

BEGIN i := top;
   WHILE i # p DO
   BEGIN (*scan linked list and find minimal key*)
      m := i; j := t[i].fol;
      WHILE j # p DO
         BEGIN IF t[j].key < t[m].key THEN m := j;
            j := t[j].fol
         END ;
      printword(t[m]);
      IF m # i THEN
         BEGIN t[m].key := t[i].key;
            t[m].first := t[i].first; t[m].last := t[i].last
         END ;
      i := t[i].fol
   END
END (*printtable*) ;

BEGIN n := 0; k1 := c1; top := p; reset(f);
   FOR i := 0 TO p DO t[i].key := free;
   letters := ["a".."z"]; letdigs := letters + ["0".."9"];
   WHILE NOT eof(f) DO
   BEGIN IF n = c4 THEN n := 0;
      n := n+1; write(n:c3);       (*next line*)
      write(" ");
      WHILE NOT eoln(f) DO
      BEGIN (*scan non-empty line*)
         IF f↑ IN letters THEN
         BEGIN k := 0;
            REPEAT IF k < c1 THEN
               BEGIN k := k+1;  a[k] := f↑;
               END ;
               write(f↑); get(f)
            UNTIL NOT (f↑ IN letdigs);
            IF k >= k1 THEN k1 := k ELSE
            REPEAT a[k1] := " ";  k1 := k1-1
            UNTIL k1 = k;
            pack(a,1,id); search;
         END ELSE
         BEGIN (*check for quote or comment*)
            IF f↑ = """" THEN
```

```
            REPEAT write(f↑); get(f)
            UNTIL f↑ = """" ELSE
         IF f↑ = "{" THEN
            REPEAT write(f↑); get(f)
            UNTIL f↑ = "}" ;
         write(f↑); get(f)
      END
   END ;
   writeln; get(f)
END ;

13: page; printtable
END .
```

12. Syntax analysis

Skeleton compiler which checks the syntax of its input text according to the following grammar. Principle is top-down, recursive descent with one symbol lookahead. (see also N.Wirth, *Algorithms + Data Structures = Programs*, Ch. 5, Prentice-Hall, Inc. 1975)

```
program =  block "." .
block =    [ "CONST" ident "=" number {"," ident "=" number} ";"]
           [ "VAR" ident {"," ident} ";"]
           { "PROCEDURE" ident ";" block ";"} statement .
statement = [ ident ":=" expression | "CALL" ident |
           "BEGIN" statement {";" statement} "END" |
           "IF" condition "THEN" statement |
           "WHILE" condition "DO" statement ] .
condition = "ODD" expression |
           expression ("="|"#"|"<"|"<="|">"|">=") expression .
expression = ["+"|"-"] term {("+"|"-") term}.
term =     factor {("*"|"/") factor}.
factor =   ident | number | "(" expression ")" .
```

```
PROGRAM PL0(input,output);
LABEL 99;

CONST norw = 11;        (*no. of reserved words*)
  txmax = 100;          (*length of identifier table*)
  nmax = 14;            (*max. no of digits in numbers*)
  al = 10;              (*length of identifiers*)
  chsetsize = 128;      (*for ASCII character set*)

TYPE symbol =
  (nul,ident,number,plus,minus,times,slash,oddsym,
   eql,neq,lss,leq,gtr,geq,lparen,rparen,comma,semicolon,
   period,becomes,beginsym,endsym,ifsym,thensym,
   whilesym,dosym,callsym,constsym,varsym,procsym);
  alfa = PACKED ARRAY [1..al] OF char;
  object = (constant,variable,prozedure);

VAR ch: char;           (*last character read*)
  sym: symbol;          (*last symbol read*)
  id: alfa;             (*last identifier read*)
  num: integer;         (*last number read*)
  cc: integer;          (*character count*)
  ll: integer;          (*line length*)
  kk: integer;
  line: ARRAY [1..81] OF char;
  a: alfa;
  word: ARRAY [1..norw] OF alfa;
  wsym: ARRAY [1..norw] OF symbol;
  ssym: ARRAY [char] OF symbol;
  table: ARRAY [0..txmax] OF
        RECORD name: alfa;
               kind: object
        END ;
```

```
PROCEDURE error(n: integer);
BEGIN writeln(" ":cc, "↑", n:2); GOTO 99
END (*error*) ;

PROCEDURE getsym;
  VAR i,j,k: integer;

  PROCEDURE getch;
  BEGIN IF cc = ll THEN
    BEGIN IF eof(input) THEN
          BEGIN write(" program incomplete"); GOTO 99
          END ;
      ll := 0; cc := 0; write(" ");
      WHILE NOT eoln(input) DO
        BEGIN ll := ll+1; read(ch); write(ch); line[ll] := ch
        END ;
      writeln; ll := ll+1; read(line[ll])
    END ;
    cc := cc+1; ch := line[cc]
  END (*getch*) ;

BEGIN (*getsym*)
  WHILE ch = " " DO getch;
  IF ch IN ["a".."z"] THEN
  BEGIN (*identifier or reserved word*)  k := 0;
    REPEAT IF k < al THEN
      BEGIN k := k+1; a[k] := ch
      END ;
      getch
    UNTIL NOT (ch IN ["a".."z","0".."9"]);
    IF k >= kk THEN kk := k ELSE
      REPEAT a[kk] := " "; kk := kk-1
      UNTIL kk = k;
    id := a; i := 1; j := norw;
    REPEAT k := (i+j) DIV 2;
      IF id <= word[k] THEN j := k-1;
      IF id >= word[k] THEN i := k+1
    UNTIL i > j;
    IF i-1 > j THEN sym := wsym[k] ELSE sym := ident
  END ELSE
  IF ch IN ["0".."9"] THEN
  BEGIN (*number*) k := 0; num := 0; sym := number;
    REPEAT num := 10*num + (ord(ch)-ord("0"));
      k := k+1; getch
    UNTIL NOT (ch IN ["0".."9"]);
    IF k > nmax THEN error(30)
  END ELSE
  IF ch = ":" THEN
  BEGIN getch;
    IF ch = "=" THEN
    BEGIN sym := becomes; getch
    END ELSE sym := nul;
  END ELSE
  IF ch = "<" THEN
  BEGIN getch;
```

```
      IF ch = "=" THEN
      BEGIN sym := leq; getch
      END ELSE sym := lss
   END ELSE
   IF ch = ">" THEN
   BEGIN getch;
      IF ch = "=" THEN
      BEGIN sym := geq; getch
      END ELSE sym := gtr
   END ELSE
   BEGIN sym := ssym[ch]; getch
   END
END (*getsym*) ;


PROCEDURE block(tx: integer);

   PROCEDURE enter(k: object);
   BEGIN (*enter object into table*)
      tx := tx + 1;
      WITH table[tx] DO
      BEGIN name := id; kind := k;
      END
   END (*enter*) ;

   FUNCTION position(id: alfa): integer;
      VAR i: integer;
   BEGIN (*find identifier id in table*)
      table[0].name := id;  i := tx;
      WHILE table[i].name # id DO  i := i-1;
      position := i
   END (*position*) ;

   PROCEDURE constdeclaration;
   BEGIN IF sym = ident THEN
      BEGIN getsym;
         IF sym = eql THEN
         BEGIN getsym;
            IF sym = number THEN
               BEGIN enter(constant); getsym
               END
            ELSE error(2)
         END ELSE error(3)
      END ELSE error(4)
   END (*constdeclaration*) ;

   PROCEDURE vardeclaration;
   BEGIN IF sym = ident THEN
         BEGIN enter(variable); getsym
         END ELSE error(4)
   END (*vardeclaration*) ;

   PROCEDURE statement;
      VAR i: integer;
```

```
  PROCEDURE expression;

    PROCEDURE term;

      PROCEDURE factor;
        VAR i: integer;
      BEGIN
        IF sym = ident THEN
        BEGIN i := position(id);
          IF i = 0 THEN error(11) ELSE
          IF table[i].kind = prozedure THEN error(21);
          getsym
        END ELSE
        IF sym = number THEN
        BEGIN getsym
        END ELSE
        IF sym = lparen THEN
        BEGIN getsym; expression;
          IF sym = rparen THEN getsym ELSE error(22)
        END
        ELSE error(23)
      END (*factor*) ;

    BEGIN (*term*) factor;
      WHILE sym IN [times,slash] DO
        BEGIN getsym; factor
        END
    END (*term*) ;

  BEGIN (*expression*)
    IF sym IN [plus,minus] THEN
      BEGIN getsym; term
      END ELSE term;
    WHILE sym IN [plus,minus] DO
      BEGIN getsym; term
      END
  END (*expression*) ;

  PROCEDURE condition;
  BEGIN
    IF sym = oddsym THEN
    BEGIN getsym; expression
    END ELSE
    BEGIN expression;
      IF NOT (sym IN [eql,neq,lss,leq,gtr,geq]) THEN
        error(20) ELSE
      BEGIN getsym; expression
      END
    END
  END (*condition*) ;

BEGIN (*statement*)
  IF sym = ident THEN
  BEGIN i := position(id);
    IF i = 0 THEN error(11) ELSE
    IF table[i].kind # variable THEN error(12);
```

```
      getsym; IF sym = becomes THEN getsym ELSE error(13);
      expression
    END ELSE
    IF sym = callsym THEN
    BEGIN getsym;
      IF sym # ident THEN error(14) ELSE
        BEGIN i := position(id);
          IF i = 0 THEN error(11) ELSE
          IF table[i].kind # prozedure THEN error(15);
          getsym
        END
    END ELSE
    IF sym = ifsym THEN
    BEGIN getsym; condition;
      IF sym = thensym THEN getsym ELSE error(16);
      statement;
    END ELSE
    IF sym = beginsym THEN
    BEGIN getsym; statement;
      WHILE sym = semicolon DO
        BEGIN getsym; statement
        END ;
      IF sym = endsym THEN getsym ELSE error(17)
    END ELSE
    IF sym = whilesym THEN
    BEGIN getsym; condition;
      IF sym = dosym THEN getsym ELSE error(18);
      statement
    END
  END (*statement*) ;

BEGIN (*block*)
  IF sym = constsym THEN
  BEGIN getsym; constdeclaration;
    WHILE sym = comma DO
      BEGIN getsym; constdeclaration
      END ;
    IF sym = semicolon THEN getsym ELSE error(5)
  END ;
  IF sym = varsym THEN
  BEGIN getsym; vardeclaration;
    WHILE sym = comma DO
      BEGIN getsym; vardeclaration
      END ;
    IF sym = semicolon THEN getsym ELSE error(5)
  END ;
  WHILE sym = procsym DO
  BEGIN getsym;
    IF sym = ident THEN
      BEGIN enter(prozedure); getsym
      END
    ELSE error(4);
    IF sym = semicolon THEN getsym ELSE error(5);
    block(tx);
```

```
    IF sym = semicolon THEN getsym ELSE error(5);
  END ;
  statement
END (*block*) ;

BEGIN (*main program*)
  FOR ch := chr(0) TO chr(chsetsize-1) DO ssym[ch] := nul;
  word[ 1] := "BEGIN     "; word[ 2] := "CALL      ";
  word[ 3] := "CONST     "; word[ 4] := "DO        ";
  word[ 5] := "END       "; word[ 6] := "IF        ";
  word[ 7] := "ODD       "; word[ 8] := "PROCEDURE ";
  word[ 9] := "THEN      "; word[10] := "VAR       ";
  word[11] := "WHILE     ";
  wsym[ 1] := beginsym; wsym[ 2] := callsym;
  wsym[ 3] := constsym; wsym[ 4] := dosym;
  wsym[ 5] := endsym;   wsym[ 6] := ifsym;
  wsym[ 7] := oddsym;   wsym[ 8] := procsym;
  wsym[ 9] := thensym;  wsym[10] := varsym;
  wsym[11] := whilesym;
  ssym["+"] := plus;    ssym["-"] := minus;
  ssym["*"] := times;   ssym["/"] := slash;
  ssym["("] := lparen;  ssym[")"] := rparen;
  ssym["="] := eql;     ssym[","] := comma;
  ssym["."] := period;  ssym["#"] := neq;
  ssym["<"] := lss;     ssym[">"] := gtr;
  ssym[";"] := semicolon;
  page(output);
  cc := 0; ll := 0; ch := " "; kk := al; getsym;
  block(0);
  IF sym # period THEN error(9);
99: writeln
END .
```

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